Trading and Market Microstructure

Research paper on

The Average Compounded Growth Rate Model



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Introduction

People tend to apply severely different approaches to achieve financial gains in the markets around the world. Some strategies are based on fundamental analysis and a belief that the intrinsic value of an asset will determine the price and forecast further returns. Others tend to apply more price-oriented strategies where history, psychology, trends and momentum are analyzed in a quantitative manner through charts. Nowadays one can find thousands of strategies over the internet that are supposed to "beat the market" and to generate higher returns. For this reason, we want to further examine and evaluate an interesting approach that is not well known to the public.

According to Mike Klein's BMW-method (2017), the compound annual growth rate (CAGR) can be used to determine whether a stock is over- or undervalued, thus whether "its price is below or above its fundamental value" (Harris, 2002), and further if a practitioner should buy or sell it. In the following essay, we shall examine the CAGR and develop a strategy that we will compare to the Dow Jones Industrial Average. The research question we want to answer is: *How does the BMW-method perform compared to the Dow Jones Industrial Average?*

There will be constructed a virtual portfolio, by gathering price data from Wharton Research Data Service, from 1986 until 2016, to see if we can use this method to gain alpha. We shall analyze the strategy, discuss the risks involved as well as the pros and cons related to the method before ending with our conclusion.

Theoretical framework

Background and context

In the fall of 2010, a member of the stock subscription service, The Motley Fool, wrote on the Fool forum that he claimed to see a pattern in the Dow Jones Industrial Average (DJIA as of now) that nobody else was paying attention to. He claimed that by observing this, he had been able to beat the DJIA consequently for more than 10 years. Even more, he used this method as his primary income during those 10 years (Klein, 2017). The method is named BMW-method after his nickname on The Motley Fool.

The hypothesis is that the DJIA, in the long term, seems to follow an exponential growth curve, which the anonymous user called the "Average CAGR curve". Of course, prices will

have bounces above and below this growth curve, however in the long-term perspective, the DJIA should move towards the curve's direction. Furthermore, he claimed that once the prices are subject to a large deviation from the curve, they tend to overshoot the Average CAGR when reverting to the mean, by continuing their "reversion path" well above the average.

This method was later introduced to single stocks by investor Mike Klein (2017). Klein claimed that this was not something particular for the DJIA, but that a company's stock could show the same behavior. He refined and developed the strategy to become more sophisticated, and we will apply his interpretations of the strategy. Further, the member of the Fool forum mentioned IBM as a perfect example where the stock was far below the Average CAGR from 1970 until 1990, before it "reverted back to the mean" and overshoot the Average CAGR from 1990 until 2010. Another example can be the CVS Health Corporation that is by year-end 2016 priced very close to the Average CAGR.

The overall concept (which we shall elaborate) of the method is hence the following:
First, we calculate the current CAGR of a specific year and then we sum it with the calculation of the CAGR for the prior years. Afterwards, we divide this sum by the specific year, t, and call this number the Average CAGR. The number, e.g. 10%, is used in two ways: First, an average CAGR of 10% tells us that the return from t=0 to our respective t, has been 10%. Second, the model says that in the future, 10% should also be the annual return of the stock. In total, that means that when the stock trades at CAGR higher than 10%, it´s overvalued and when it trades at less than 10%, it´s undervalued, and that is the core of our model.

Data collection

We gathered all the data necessary for our work in the Wharton Research Data Service (WRDS) that NHH provided us. Our objective was to gather price data for each stock entering or leaving the DJIA in our set period.

First, we collected data about the historical components of the DJIA through easy-accessible information on the internet. Then, we used these data to create a list of all the firms that entered and left the DJIA from 1986 until 2016 followed with the respective enter and exit

dates. In total that summed up to 50 companies. When a company changed its name due to a merger, we continued our count with the new company name.

Second, we used the Wharton Database to download daily prices of each stock ranging from 1986 until 2016. We collected data not only for the permanence period in the index, but also for the 11 years previous to the entrance date. This data, in fact, were necessary to calculate the Current CAGR and the Average CAGR for each stock. The first price in our database, thus, was the one in the first trading day of the 11^{th} year previous to the entrance date, and the last price was the one in the exact date in which the firm left the DJIA.

The two following figures show an overview of what our portfolio looked like during the data collection phase of the work. Figure 1 displays the complete list of companies taking part in the Dow Jones from 1986 to 2016, with the respective entry and exit dates. We created four columns since a company, (e.g. Chevron in our case) may exit and then reenter the DJIA within our period.

In Figure 2, instead, we show an example of what a single-company database looked like. For each company, we created a table with the company name, (keeping the old historical names and changing it whenever the company entered a merger or changed its name), stock price and a series of useful measures for the valuation and choice of the stock that we will further explain in the following section.

	We conduct investment and portfolio decisions from 01/01/97 to 31/12/16; we need data starting from 01/01/86.										
	We have to make sure that the END_1/2-Days are TRADING DAYS, if not, we leave the date in THIS file like it is and just use the next possible trading day BEFORE in the real data sheet.										
	Usually, the day of change given on the Wikipedia site is the Monday, so the end dates we refer to here are Sundays (i.e. not trading days).										
	However, we leave these data in THIS SHEET so we can match outgoing and ingoing stocks.										
	The respective portfolio decision is made only on 01/01/XX, we sell stocks when they leave the DJIA, but do not buy new ones when they come in intra-year.										
	Do not confuse: AT&T Corporation left the DJIA on 07/04/04. However, SBC Communications Inc. was renamed AT&T Inc. on 21/11/05 after acquiring the original AT&T Corporation.										
	Relevant for data extraction are START_DATE and END_DATE, only one stock has two STARTS and ENDS, as it re-enters the index.										
NUMBER	NAME	NAME HISTORY	TICKER	START_1	END_1	START_2	END_2	START_DATE	END_DATE	START_DATE_2	END_DATE_2
1	AlliedSignal Incorporated	merged to Honeywell International (27/01/03)		01.01.1997	18.02.2008			01.01.1997	18.02.2008		
2	Aluminum Company of America	to Alcoa Inc. (01/11/99)		01.01.1997	25.09.2013			01.01.1997	25.09.2013		
3	American Express Company			01.01.1997				01.01.1997	31.12.2016		
4	American International Group Inc.			08.04.2004	21.09.2008			08.04.2004	21.09.2008		
5	Apple Inc.			19.03.2015				19.03.2015	31.12.2016		
6	AT&T Corporation			01.01.1997	07.04.2004			01.01.1997	07.04.2004		
7	Bank of America Corporation			19.02.2008	25.09.2013			19.02.2008	25.09.2013		
8	Betlehem Steel Corporation (ignored)			01.01.1997	16.03.1997			01.01.1997	16.03.1997		
9	Caterpillar Inc.			01.01.1997				01.01.1997	31.12.2016		
10	Chevron Corporation			01.01.1997	31.10.1999	19.02.2008		01.01.1997	31.10.1999	19.02.2008	31.12.2016
11	Cisco Systems, Inc.			08.06.2009				08.06.2009	31.12.2016		
12	E.I. du Pont de Nemours & Company			01.01.1997				01.01.1997	31.12.2016		
13	Eastman Kodak Company (left out)			01.01.1997	07.04.2004			01.01.1997	07.04.2004		
14	Exxon Corporation	merged to Exxon Mobil Corporation (27/01/03)		01.01.1997				01.01.1997	31.12.2016		
15	General Electric Company			01.01.1997				01.01.1997	31.12.2016		
16	General Motors Corporation			01.01.1997	07.06.2009			01.01.1997	07.06.2009		
17	Goodyear Tire and Rubber Company			01.01.1997	31.10.1999			01.01.1997	31.10.1999		
18	Hewlett-Packard Company			17.03.1997	25.09.2013			17.03.1997	25.09.2013		
19	Intel Corporation			01.11.1999				01.11.1999	31.12.2016		
20	International Business Machines Corporation			01.01.1997				01.01.1997	31.12.2016		
21	International Paper Company			01.01.1997	07.04.2004			01.01.1997	07.04.2004		
22	J.P. Morgan & Company (*)	to JPMorgan Chase & Co. (27/01/03)		01.01.1997				01.01.1997	31.12.2016		
23	Johnson & Johnson			17.03.1997				17.03.1997	31.12.2016		
24	Kraft Foods Inc.			22.09.2008	23.09.2012			22.09.2008	23.09.2012		
25	McDonald's Corporation			01.01.1997				01.01.1997	31.12.2016		
26	Merck & Co., Inc.			01.01.1997				01.01.1997	31.12.2016		
27	Microsoft Corporation			01.11.1999				01.11.1999	31.12.2016		
28	Minnesota Mining & Manufacturing Company	to 3M Company (27/01/03)		01.01.1997				01.01.1997	31.12.2016		
29	Nike, Inc.			26.09.2013				26.09.2013	31.12.2016		
30	Pfizer Inc.			08.04.2004				08.04.2004	31.12.2016		
31	Philip Morris Companies Inc.	to Altria Group, Incorporated (27/01/03)		01.01.1997	18.02.2008			01.01.1997	18.02.2008		
32	SBC Communications Inc. (left out)	to AT&T Inc. (21/11/05; after acquiring the original AT&T)		01.11.1999	18.03.2015			01.11.1999	18.03.2015		
33	Sears Roebuck & Company (left out)			01.01.1997	31.10.1999			01.01.1997	31.10.1999		
34	Texaco Incorporated (ignored)			01.01.1997	16.03.1997			01.01.1997	16.03.1997		
35	The Boeing Company			01.01.1997				01.01.1997	31.12.2016		
36	The Coca-Cola Company			01.01.1997				01.01.1997	31.12.2016		
37	The Goldman Sachs Group, Inc.			26.09.2013				26.09.2013	31.12.2016		
38	The Home Depot, Inc.			01.11.1999				01.11.1999	31.12.2016		
39	The Procter & Gamble Company			01.01.1997				01.01.1997	31.12.2016		
40	The Travelers Companies, Inc. (left out)			08.06.2009				08.06.2009	31.12.2016		
41	The Walt Disney Company			01.01.1997				01.01.1997	31.12.2016		
42	Travelers Inc.	merged to Citigroup Inc. (01/11/99)		17.03.1997	07.06.2009			17.03.1997	07.06.2009		
43	Union Carbide Corporation (split history unsure)			01.01.1997	31.10.1999			01.01.1997	31.10.1999		
44	United Technologies Corporation			01.01.1997				01.01.1997	31.12.2016		
45	UnitedHealth Group Incorporated			24.09.2012				24.09.2012	31.12.2016		
46	Verizon Communications Inc.			08.04.2004				08.04.2004	31.12.2016		
47	Visa Inc.			26.09.2013				26.09.2013	31.12.2016		
48	Wal-Mart Stores, Inc.			17.03.1997				17.03.1997	31.12.2016		
49	Westinghouse Electric Corporation (left out)			01.01.1997	16.03.1997			01.01.1997	16.03.1997		
50	Woolworth's (left out)			01.01.1997	16.03.1997			01.01.1997	16.03.1997		

Figure 1: Tracking method of the stocks within DJIA from 1986 until 2016

	1	SELLING CRITERION			LEAVES INDEX BEFORE END									
	0,525	BUYING CRITERION			ENTERED INDEX IN YEAR BEFORE THIS DATE									
	PLEASE SELECT OR CHANGE ALL CRITERIA AT THE BOTTOM OF THE FILE IF YOU PLAY AROUND WITH IT!!!													
							price %	C_CAGR	CUM_CC	A_CAGR	RATIO	CRI	INV?	PORT?
1	10145	01/02/1986	ALD	ALLIED SIGNAL INC	46,875	46,875		46,875				1		
2	10145	12/31/1986	ALD	ALLIED SIGNAL INC	40,125	40,125	-0,144	-0,144	-0,144	-0,144	1	1		
3	10145	12/31/1987	ALD	ALLIED SIGNAL INC	28,25	28,25	-0,295950156	-0,223683913	-0,367684	-0,183842	1,2167	1		
4	10145	12/30/1988	ALD	ALLIED SIGNAL INC	32,5	32,5	0,150442478	-0,114923737	-0,482608	-0,160869	0,7144	1		
5	10145	12/29/1989	ALD	ALLIED SIGNAL INC	34,875	34,875	0,073076923	-0,07126196	-0,55387	-0,138467	0,5146	1		
6	10145	12/31/1990	ALD	ALLIED SIGNAL INC	27	27	-0,225806452	-0,104461015	-0,658331	-0,131666	0,7934	1		
7	10145	12/31/1991	ALD	ALLIED SIGNAL INC	43,875	43,875	0,625	-0,010962766	-0,669293	-0,111549	0,0983	1		
8	10145	12/31/1992	ALD	ALLIED SIGNAL INC	60,5	60,5	0,378917379	0,037123762	-0,63217	-0,09031	-0,4111	1		
9	10145	12/31/1993	ALD	ALLIED SIGNAL INC	79	79	0,305785124	0,06742096	-0,564749	-0,070594	-0,9551	1		
10	10145	12/30/1994	ALD	ALLIED SIGNAL INC	34	**68**	-0,139240506	0,042202136	-0,522547	-0,058061	-0,7269	1		
11	10145	12/29/1995	ALD	ALLIED SIGNAL INC	47,5	**95**	0,397058824	0,073193991	-0,449353	-0,044935	-1,6289	1		
12	10145	12/31/1996	ALD	ALLIED SIGNAL INC	67	**134**	0,410526316	0,100194351	-0,349158	-0,031742	-3,1566	1	0	0
13	10145	12/31/1997	ALD	ALLIED SIGNAL INC	38,8125	**155,25**	0,15858209	0,104945508	-0,244213	-0,020351	-5,1568	1	0	0
14	10145	12/31/1998	ALD	ALLIED SIGNAL INC	44,3125	**177,25**	0,141706924	0,107730792	-0,136482	-0,010499	-10,261	1	0	0
15	10145	12/31/1999	HON	HONEYWELL INTERNAT	57,6875	**230,75**	0,301833568	0,120580054	-0,015902	-0,001136	-106,16	1	0	0
16	10145	12/29/2000	HON	HONEYWELL INTERNAT	47,3125	**189,25**	-0,179848321	0,097504494	0,0816027	0,00544	17,923	1	0	0
17	10145	12/31/2001	HON	HONEYWELL INTERNAT	33,82	**135,28**	-0,285178336	0,068484606	0,1500873	0,00938	7,3008	1	0	0
18	10145	12/31/2002	HON	HONEYWELL INTERNAT	24	**96**	-0,290360733	0,043070173	0,1931574	0,011362	3,7907	1	0	0
19	10145	12/31/2003	HON	HONEYWELL INTERNAT	33,43	**133,72**	0,392916667	0,059966035	0,2531235	0,014062	4,2643	1	0	0
20	10145	12/31/2004	HON	HONEYWELL INTERNAT	35,41	**141,64**	0,059228238	0,059927191	0,3130507	0,016476	3,6372	1	0	0
21	10145	12/30/2005	HON	HONEYWELL INTERNAT	37,25	**149**	0,051962722	0,059527539	0,3725782	0,018629	3,1954	1	0	0
22	10145	12/29/2006	HON	HONEYWELL INTERNAT	45,24	**180,96**	0,214496644	0,066437235	0,4390154	0,020905	3,178	1	0	0
23	10145	12/31/2007	HON	HONEYWELL INTERNAT	61,57	**246,28**	0,360963749	0,07832444	0,5173399	0,023515	3,3308	1	0	0
	10145	02/15/2008	HON	HONEYWELL INTERNATI	56,04	**224,16**	-0,089816469							

Figure 2: Data collection

Variables and statistical measures

We will now give a detailed explanation of some variables and statistical measurements used in the further discussion.

The Current CAGR

The first, and most important variable used for developing and analyzing our strategy is the Compounded Annual Growth Rate (CAGR).

$$Current\ CAGR_t = \sqrt[t]{\frac{stock\ price_t}{stock\ price_0}} - 1 \qquad (1)$$

The Current CAGR is a term used to describe the geometric progression ratio or exponential growth rate which gives an average or mean performance of an investment. The CAGR assumes that an asset, cash flow or some other random variable grows at a constant rate of return compounded over a sample period. The term has no profound ground in accounting or financial theory, but is often used as a representational figure of the performance of a variable where one neglects the historical volatility of that variable. In such, the term is useful when comparing growth rates of companies in the same industry.

The CAGR can therefore be used to analyze isolated segments within a business, comparing segments business to business or comparing the expected returns for two investments such as a saving account and a stock portfolio. However, a drawdown to the term is that it assumes

constant and steady growth rate of an investment, which happens rarely. Another limitation is that CAGR is solely based on historical prices as a variable.

In several formulas, the variable **t** is present. In our model, we only use one price observation per year, so **t** stands for the yearly observation of the stock´s price. Therefore, t=0 refers to the first trading day of 1986, but from there we use the last trading day of each respective year as trading day. This means that t=1 is the last trading day of 1987 and so on.

The Average CAGR

$$Average\ CAGR_t = \frac{\sum_{t=1}^{t} Current\ CAGR_t}{t} \qquad (2)$$

The Average CAGR is a measure that sums the annual growth rates for every historical year t, divided by t. The denominator is the last trading day of the year. Based upon the historical Average CAGR for a stock, Klein (2017) argues that the ratio can be used to predict the future growth rate.

The Selection Ratio

$$Selection\ Ratio = \frac{Current\ CAGR_t}{Average\ CAGR_t} \qquad (3)$$

The third important variable is the Selection Ratio. It is used to determine under- or overvalued stocks based on the earlier mentioned CAGR-values. If a stock trades at Current CAGR of 6% and has an Average CAGR of 10%, the Selection Ratio will be 0.6. This relative measurement will be vital when executing trades based on our strategy.

The Ex post Sharpe Ratio

The Ex post Sharpe Ratio, after the great economist William Sharpe, characterizes how well the return of an asset compensates the investor for the risk taken.

$$Sharpe\ Ratio = \frac{Rp - Rf}{\sigma p} \qquad (4)$$

Even though the Sharpe Ratio normally uses a risk-free rate as a benchmark, for this matter, we will use the DJIA rate as a comparison. This ratio can be named differently based upon different sources ("Information ratio", "Ex post Sharp Ratio" etc.) (Goodwin, 1998). The Ex post Sharpe Ratio uses the same equation as the traditional Sharpe Ratio, but with realized

returns of the asset and benchmark rather than expected returns. “The ratio indicates the historic average differential return per unit of historic variability of the differential return” (William F. Sharpe, 1994).

$$Ex\ post\ SR = \frac{Rp - Rb}{\sigma(Rp - Rb)} \quad (5)$$

Cumulative Return

The cumulative return is the aggregate amount that the portfolio, or the index, has gained or lost over the investing period, independent of the time involved (Investopedia, 2017). The cumulative return is presented as a percentage and will be used for calculation purposes in our model.

$$Cumulative\ Return = \frac{Current\ Price - Initial\ Price}{Initial\ Price} \quad (6)$$

Geometric Excess Return

“Geometric Excess Return (GER) looks at the added value we get by investing in the portfolio, relative to the benchmark. In other words, how much more money do I have than I would have had if I were invested in the benchmark” (Hens & Zaegel, 2017). We calculate the return by the following formula:

$$GER = \frac{BMW\ cumulative\ return - DOW\ cumulative\ return}{1 + DOW\ cumulative\ return} \quad (7)$$

Arithmetic Excess Return

Arithmetic Excess Return is simply the difference between the BMW cumulative return and the DJIA cumulative return. Although arithmetic excess returns are easier to use and, to some more intuitive, geometric excess returns exhibit proper proportionality, convertibility, and compound ability.

The Modigliani measure

The Modigliani – Modigliani measure, also called Modigliani risk-adjusted performance measure or just "M2", is a measure of the risk-adjusted returns of the investment portfolio. It measures the returns of the portfolio, adjusted for the risk of the portfolio relative to a benchmark. In this case the benchmark is the DJIA. It is derived and to some extend similar

to the Sharpe Ratio, but it has the significant advantage of being in units of percentage return which makes it more intuitive to interpret (F. Modigliani, 1997).

Trading Setup

We want to use and analyze a refined version of the BMW-method by tracking the DJIA from 1986 until 2016. In this portfolio, we shall follow each company in the DJIA and only buy when the Average CAGR tells us to buy. We will only consider stocks within the DJIA to better examine if one can make a concentrated portfolio based upon an index and compare their performances. The limitations of the trading setup and the strategy adaptations were made due to practical reasons and given our time constraint for the assignment.

Buying

The strategy for when to execute a buy transaction is based upon the Current CAGR, the Average CAGR and the Selection Ratio. The following formula conducts the buy signal:

$$Buy\ when\ =\ Current\ CAGR\ <\ (Average\ CAGR\ x\ Selection\ Ratio) \quad (8)$$

In practical terms, this means that we buy a stock when the stock price is below the Average CAGR multiplied by the Selection Ratio. This, based upon the BMW-method, will make us able to buy statistically undervalued stocks.

Selling

The strategy for when to execute a sell transaction is also based upon the Current CAGR, the Average CAGR and the Selection Ratio. We have the following sell signals:

$$Sell\ when\ =\ Current\ CAGR >\ Average\ CAGR = Selection\ ratio > 1 \quad (9)$$

Or:

$$Sell\ when\ =\ Stock\ leaves\ the\ DJIA$$

Consequently, we sell when the Selection Ratio is more than 1. When the stock reaches a Selection Ratio of 1, in fact, the stock should be fairly valued according to our strategy. Even though the BMW-method and the implications by Mike Klein included an emotional and behavioral aspect by introducing the "overshooting phenomenon", we want our strategy to be as rational as possible, and we want to own only undervalued stocks. Therefore, we refine the

strategy to never own stocks with a Selection Ratio that exceeds 1.

Another important aspect is how much capital that should be allocated to each stock. We were thinking about weighing the portfolio with the Selection Ratio as a determinant. This seemed rational and we thought that owning relatively more of relatively cheaper stocks would be a good idea. Nevertheless, this weighing can make our portfolio very unbalanced, and we can end up buying increasingly of companies heading for chapter 11. Therefore, and also due to simplicity, we have chosen to weigh every stock in the portfolio equal.

Enter and leave

We enter a stock that has fulfilled our buying criterion at 31.12.xx every year. We therefore rebalance at year-end. However, when a stock was removed from the DJIA-index, the stock was sold instantly. For the remaining of the year, the cash earned from the removed stock was held to year-end before a rebalance. This limitation was made due to practical reasons and to make consistent year over year comparison.

Other limitations

We do not include interest in our model. Also, because both the BMW-method and the DJIA is based upon price only, we have excluded dividends from the analysis.

Discussion

Introducing the portfolio

The main objective of this paper was to construct a portfolio, based on the BMW-method, and study how the strategy performed compared to the Dow Jones Industrial Average. By creating a profound model, we were able to abstract some interesting findings.

Plug in the Selection_Ratio here:	>>> 0,525 <<<
Buy Stock if both conditions are fulfilled:	
1. Current_CAGR < Average_CAGR x Selection_Ratio	
2. Average_CAGR > 0	
Sell Stock if Current_CAGR > Average_CAGR or when leaving the Index	
From Jan 1997 to Dec 2016 (20 years), the model gives	
BMW Cumulative Return:	853,7%
BMW Compound Annual Growth Rate:	11,9%
Dow Jones Cumulative Return:	394,2%
Dow Jones Compound Annual Growth Rate:	8,3%
Against the Dow Jones as benchmark, the model gives	
BMW Geometric Excess Return:	93,0%
BMW Geometric Mean Excess Return:	3,3%
BMW Arithmetic Excess Return:	459,5%
BMW Arithmetic Mean Excess Return:	5,8%
For Statistical Risk Analysis, the model gives	
Sample Variance (annual return BMW):	0,08
Sample Variance (annual return Dow Jones):	0,02
Sample Correlation Coefficient (Pearson's R):	0,46
Modigliani-Modigliani Measure:	0,07
Ex Post Sharpe-Ratio (1994; Benchmark Dow Jones):	0,23

Year	Stocks	BMW %	DOW %	Excess pp
1997	0	0,000	0,226	-0,226
1998	1	0,845	0,161	0,684
1999	1	0,681	0,252	0,429
2000	2	0,568	-0,062	0,629
2001	4	0,018	0,071	-0,053
2002	6	-0,111	0,168	-0,279
2003	8	0,245	0,253	-0,008
2004	8	0,032	0,032	0,000
2005	7	-0,067	-0,006	-0,061
2006	7	0,285	0,163	0,122
2007	7	-0,085	0,064	-0,149
2008	8	-0,368	-0,338	-0,030
2009	13	0,202	0,188	0,014
2010	12	0,066	0,110	-0,044
2011	12	-0,085	0,053	-0,138
2012	11	0,240	0,073	0,167
2013	12	0,251	0,265	-0,014
2014	8	0,163	0,075	0,088
2015	8	0,074	-0,023	0,097
2016	10	0,072	0,134	-0,062
Total Stocks	145	BMW Outperforms Dow Jones		
Ø Stocks	7,3	for the chosen Selection_Ratio!		

Figure 3: Investing portfolio

Figure 3 illustrates a snapshot of our investing portfolio. Our model is dynamic as it adapts to the input-variables executed by the user. It is possible to change the Selection Ratio in the upper-left part of the portfolio, at any time, and the portfolio's variables will automatically adapt.

The left part of visualized in figure 3 presents the statistical measurements used to analyze the performance. These measurements are divided into 4 tables: Starting from the top, the first table, "Plug in the Selection_Ratio here" shows the user a simple description of the portfolio's buying and selling criteria, together with the specific Selection Ratio chosen.

The second table gives an overview of what the portfolio and the index can give to the investor for any determined Selection Ratio. Specifically, we computed the Cumulative Return and the CAGR to compare our numbers against the DJIA.

The third table illustrates the actual excess returns provided by the portfolio, with a specific Selection Ratio. The excess return is calculated with the Geometric Excess Return and the Arithmetic Excess Return.

The last table shows statistical risk analysis measures such as sample variance. Further, the Modigliani measure and the Ex post Sharp Ratio will illustrate the return compared to the risk taken.

The right part of figure 3 is the descriptive part. Here the reader can distinguish 3 variables: years (time), stocks (number of stocks) and the annual growth rate in the portfolio, the DJIA and the excess return between the two.

The year variable is a very important variable in our portfolio when connected to the number of stocks. The investing period is 20 years, from 1997 to 2016 and, at the end of each year, the portfolio is rebalanced eliminating stocks falling outside the Selection Ratio or outside the DJIA.

The third important variable is the annual growth rates of the portfolio (4^{th} column) and the DJIA (5^{th} column). We computed the growth rates by simply summing the growth of the single stocks included in the portfolio or in the index in a specific year and dividing by the number of stocks in the portfolio in that specific year. Finally, we reported the excess annual growth rate of the portfolio with respect to the index (6^{th} column).

Analyzing our results

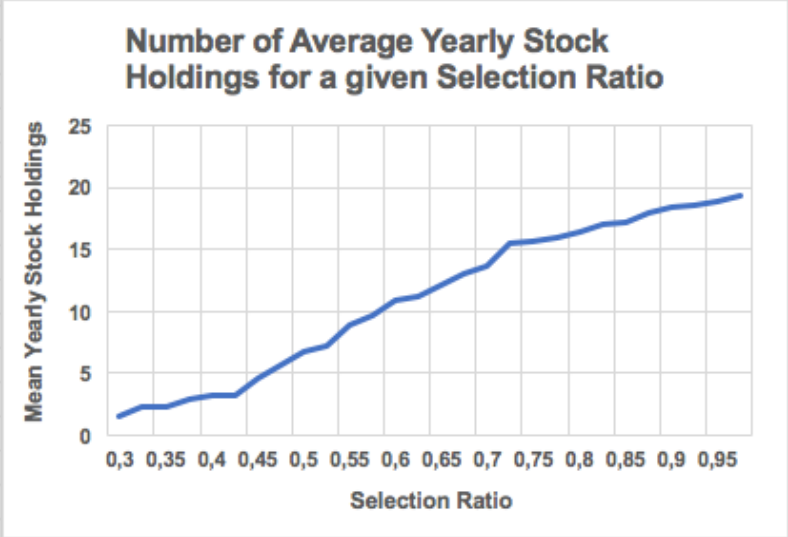
First, our results show that when using a Selection Ratio of 0.525, our portfolio had a Cumulative Return of **853.7 %** while the DJIA Cumulative Return was "only" **394.2 %.** Furthermore, our CAGR was **11.9 %** against DJIA CAGR of **8.3 %.**

Criterion	Average Positions	Total Positions	Standard variation (BMW)	PearsonR	The Modigliani	Sharpe Ratio	Geoemtric Mean Excess Return	Geometric Excess Return	CAGR (BMW)
0,3	1,45	29	0,0601	0,5432	0,0108	-0,3586	-0,0886	-0,8437	-0,013
0,325	2,25	45	0,0553	0,6206	0,0187	-0,3316	-0,0758	-0,7934	0,001
0,35	2,25	45	0,0553	0,6206	0,0187	-0,3316	-0,0758	-0,7934	0,001
0,375	2,8	56	0,0541	0,6333	0,0230	-0,3706	-0,0703	-0,767	0,007
0,4	3,15	63	0,0510	0,6318	0,0172	-0,3706	-0,0789	-0,8066	-0,002
0,425	3,2	64	0,0530	0,6484	0,0117	-0,4190	-0,0875	-0,8399	-0,012
0,45	4,5	90	0,0972	0,6141	0,0518	0,0933	-0,0094	-0,1721	0,073
0,475	5,6	112	0,0902	0,5095	0,0654	0,1881	0,0199	0,4835	0,105
0,5	6,75	135	0,0884	0,5117	0,0706	0,2281	0,0294	0,7854	0,115
0,525	7,25	145	0,0818	0,4617	0,0733	0,2296	0,0334	0,9299	0,119
0,55	8,8	176	0,0578	0,5228	0,0752	0,1819	0,0228	0,5692	0,108
0,575	9,6	192	0,0261	0,6461	0,0898	0,0917	0,0089	0,1931	0,093
0,6	10,9	218	0,0258	0,6696	0,0881	0,0738	0,0063	0,1349	0,09
0,625	11,2	224	0,0255	0,6723	0,0885	0,0732	0,0063	0,1344	0,09
0,65	12,1	242	0,0259	0,6635	0,0847	0,0441	0,003	0,0611	0,086
0,675	13,05	261	0,0229	0,7628	0,0819	-0,0356	-0,0042	-0,081	0,079
0,7	13,7	274	0,0271	0,7452	0,0810	0,0289	0,0007	0,0138	0,084
0,725	15,5	310	0,0225	0,7677	0,0851	-0,0095	-0,0015	-0,0286	0,082
0,75	15,65	313	0,0223	0,7668	0,0855	-0,0089	-0,0013	-0,0253	0,082
0,775	15,95	319	0,0225	0,7426	0,0844	-0,0165	-0,0021	-0,0403	0,081
0,8	16,4	328	0,0197	0,7724	0,0842	-0,0816	-0,0066	-0,1243	0,076
0,825	16,95	339	0,0206	0,7677	0,0784	-0,1220	-0,0109	-0,1963	0,071
0,85	17,1	342	0,0200	0,7687	0,0778	-0,1441	-0,0125	-0,2217	0,07
0,875	18	360	0,0195	0,7769	0,0781	-0,1527	-0,0127	-0,2263	0,069
0,9	18,4	368	0,0201	0,7844	0,0761	-0,1648	-0,014	-0,2461	0,068
0,925	18,5	370	0,0202	0,7949	0,0747	-0,1817	-0,0152	-0,2633	0,067
0,95	18,8	376	0,0207	0,8208	0,0723	-0,2124	-0,0168	-0,2876	0,065
0,975	19,3	386	0,0216	0,8229	0,0683	-0,2406	-0,0195	-0,326	0,062

Figure 4: Measurements for different Selection Ratios

As a way of interpreting the full impact of our model and results, we divided our analysis in different parts so that it´s possible to both see the isolated effect of the variables as well as the total effect on the portfolio.

In figure 4, the first measurement is "Average Positions", which tells us how many stocks on average each selection criteria gave from 1986 until 1996. Our top selection criteria resulted in an average position of 7.3 while having a total position of 145. As we can see, the less strict the selection threshold, the more stocks end up in the portfolio throughout the investment period. Notice that the relationship is likely to be linear, underlining the argument that trading at lower ratios of Current CAGR/ Average CAGR is less likely than trading at higher ratios.



Year	Stocks	BMW %	DOW %	Excess pp
1997	0	0,000	0,226	-0,226
1998	1	0,845	0,161	0,684
1999	1	0,681	0,252	0,429
2000	2	0,568	-0,062	0,629
2001	4	0,018	0,071	-0,053
2002	6	-0,111	0,168	-0,279
2003	8	0,245	0,253	-0,008
2004	8	0,032	0,032	0,000
2005	7	-0,067	-0,006	-0,061
2006	7	0,285	0,163	0,122
2007	7	-0,085	0,064	-0,149
2008	8	-0,368	-0,338	-0,030
2009	13	0,202	0,188	0,014
2010	12	0,066	0,110	-0,044
2011	12	-0,085	0,053	-0,138
2012	11	0,240	0,073	0,167
2013	12	0,251	0,265	-0,014
2014	8	0,163	0,075	0,088
2015	8	0,074	-0,023	0,097
2016	10	0,072	0,134	-0,062

Figure 5: Number of Average Yearly Stock Holdings for a given Selection Ratio together with the right side of the portfolio screen

Another aspect which is interesting to discuss, is the noticeable correlation between how the numbers of stocks in our portfolio corresponded with the impact from economics and market turbulence. By choosing the Selection Ratio to be 0.525 we notice that from 2000 to 2003 the portfolio increased from 2 to 8 stocks, and from 2008 to 2013, it increased from 8 to 12 stocks. These years precisely coincide to the "dot-com bubble" collapse occurred between years 2000 and 2002 and the "financial crisis" in 2008. This phenomenon is not surprising. In fact, it is perfectly in line with our BMW investing method, where the main idea is to pick stocks that are growing in the long run (Average CAGR > 0), but that are currently poorly performing (Current CAGR < Average CAGR*Selection Ratio). As we can see from figure 5, the magnitude of this poor performance is determined by the Selection Ratio we have selected.

The Geometric Excess Return

Since we wanted to know if we outperformed the DJIA, we used the Geometric Excess Return. Figure 6 below shows the results for different computed Selection Ratios.

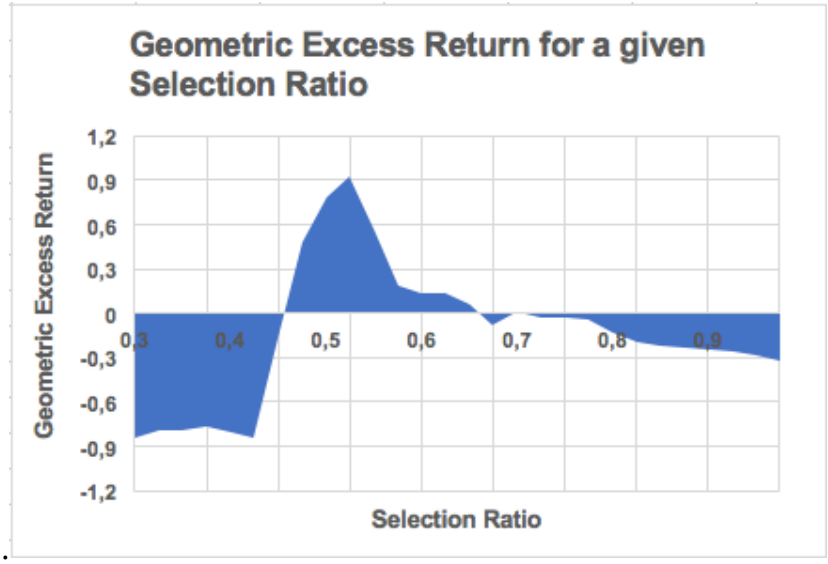


Figure 6: Geometric Excess Return

The Geometric Excess Return was clearly highest when using a Selection Ratio of about 0.525. It becomes clear that choosing the "right" Selection Ratio is crucial for our method to work well. The interval of the Selection Ratios that gives excess returns is rather small. For example, when the Selection Ratio are changed from 0.525 to 0.1, the cumulative return for our model is –2.9%, being beaten by the DJIA at 394.2%. Also, we can see that the computed Selection Ratio is dramatically sensitive for performing great and close to disaster. While the Selection Ratio of 0.525 heavily beats the DJIA, changing it from 0.43 to 0.42, the portfolio's Cumulative Return changes from 165.5 % to –20.9%. This clearly shows that while our strategy might outperform the DJIA at some ratios, it´s to some extent totally random and it´s debatable if one can use the method as an investment strategy.

Plug in the Selection_Ratio here:	>>>	0,43	<<<
Buy Stock if both conditions are fulfilled:			
1. Current_CAGR < Average_CAGR x Selection_Ratio			
2. Average_CAGR > 0			
Sell Stock if Current_CAGR > Average_CAGR or when leaving the Index			
From Jan 1997 to Dec 2016 (20 years), the model gives			
BMW Cumulative Return:		165,6%	
BMW Compound Annual Growth Rate:		5,0%	
Dow Jones Cumulative Return:		394,2%	
Dow Jones Compound Annual Growth Rate:		8,3%	
Against the Dow Jones as benchmark, the model gives			
BMW Geometric Excess Return:		-46,3%	
BMW Geometric Mean Excess Return:		-3,1%	
BMW Arithmetic Excess Return:		-228,6%	
BMW Arithmetic Mean Excess Return:		0,6%	
For Statistical Risk Analysis, the model gives			
Sample Variance (annual return BMW):		0,10	
Sample Variance (annual return Dow Jones):		0,02	
Sample Correlation Coefficient (Pearson's R):		0,61	
Modigliani-Modigliani Measure:		0,04	
Ex Post Sharpe-Ratio (1994; Benchmark Dow Jones):		0,02	

Year	Stocks	BMW %	DOW %	Excess pp
1997	0	0,000	0,226	-0,226
1998	1	0,845	0,161	0,684
1999	1	0,681	0,252	0,429
2000	1	0,222	-0,062	0,284
2001	1	-0,011	0,071	-0,082
2002	2	-0,321	0,168	-0,489
2003	4	0,398	0,253	0,145
2004	3	0,018	0,032	-0,013
2005	4	-0,125	-0,006	-0,119
2006	4	0,329	0,163	0,166
2007	4	-0,106	0,064	-0,170
2008	4	-0,600	-0,338	-0,261
2009	8	0,171	0,188	-0,017
2010	7	0,047	0,110	-0,063
2011	7	-0,171	0,053	-0,224
2012	6	0,262	0,073	0,189
2013	7	0,157	0,265	-0,108
2014	3	0,171	0,075	0,096
2015	3	-0,064	-0,023	-0,041
2016	4	0,084	0,134	-0,050
Total Stocks	74	BMW Underperforms Dow Jones		
Ø Stocks	3,7	for the chosen Selection_Ratio!		

Figure 7: Selection Ratio 0.43

Plug in the Selection_Ratio here:	>>>	0,42	<<<
Buy Stock if both conditions are fulfilled:			
1. Current_CAGR < Average_CAGR x Selection_Ratio			
2. Average_CAGR > 0			
Sell Stock if Current_CAGR > Average_CAGR or when leaving the Index			
From Jan 1997 to Dec 2016 (20 years), the model gives			
BMW Cumulative Return:	-20,9%		
BMW Compound Annual Growth Rate:	-1,2%		
Dow Jones Cumulative Return:	394,2%		
Dow Jones Compound Annual Growth Rate:	8,3%		
Against the Dow Jones as benchmark, the model gives			
BMW Geometric Excess Return:	-84,0%		
BMW Geometric Mean Excess Return:	-8,8%		
BMW Arithmetic Excess Return:	-415,1%		
BMW Arithmetic Mean Excess Return:	-7,4%		
For Statistical Risk Analysis, the model gives			
Sample Variance (annual return BMW):	0,05		
Sample Variance (annual return Dow Jones):	0,02		
Sample Correlation Coefficient (Pearson's R):	0,65		
Modigliani-Modigliani Measure:	0,01		
Ex Post Sharpe-Ratio (1994; Benchmark Dow Jones):	-0,42		

Year	Stocks	BMW %	DOW %	Excess pp
1997	0	0,000	0,226	-0,226
1998	0	0,000	0,161	-0,161
1999	0	0,000	0,252	-0,252
2000	0	0,000	-0,062	0,062
2001	0	0,000	0,071	-0,071
2002	1	-0,340	0,168	-0,507
2003	3	0,404	0,253	0,151
2004	2	0,031	0,032	0,000
2005	3	-0,169	-0,006	-0,163
2006	3	0,390	0,163	0,227
2007	3	0,016	0,064	-0,049
2008	4	-0,600	-0,338	-0,261
2009	8	0,171	0,188	-0,017
2010	7	0,047	0,110	-0,063
2011	7	-0,171	0,053	-0,224
2012	6	0,262	0,073	0,189
2013	7	0,157	0,265	-0,108
2014	3	0,171	0,075	0,096
2015	3	-0,064	-0,023	-0,041
2016	4	0,084	0,134	-0,050
Total Stocks	64	BMW Underperforms Dow Jones		
Ø Stocks	3,2	for the chosen Selection_Ratio!		

Figure 8: Selection Ratio 0.42

Variance, Excess Return and Risk

However, while excess return is important when analyzing an investment strategy against another, it´s crucial that one also examine the risk taken. Hence, we have calculated the variance between our portfolio for any given Selection Ratio and the excess return with respect to the DJIA. Our results tell two things: There is a high correlation between the portfolio sample variance of yearly returns and the Geometric Excess Return (against the DJIA), and that excess return probably comes at a cost. The referred cost here is increased risk involved with our strategy, which is in line with general finance theory which states that there is a strong correlation between return and risk.

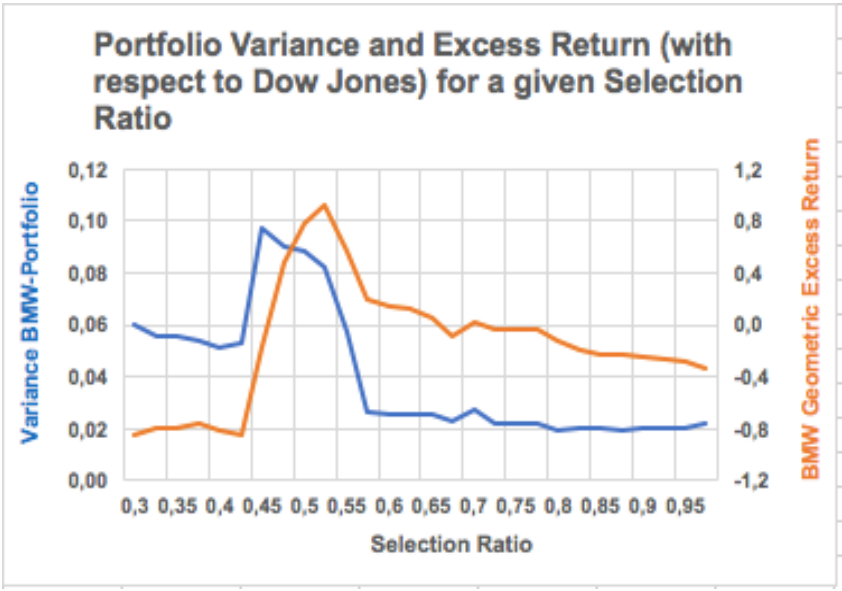


Figure 9: Portfolio Variance and Excess Return

It's interesting to compare the variance of the portfolio against our benchmark. For the higher Selection Ratio, the more stocks make it into the portfolio, resulting in a higher correlation coefficient (yearly returns' co-movement with the Index). The sample variance of yearly returns of the portfolio can be seen as a simplified inverse-U relationship with respect to the Selection Ratio. Notice that the moderate Selection Ratios, from 0.3 to 0.6, seem to produce the highest variance portfolios, a relationship that somewhat corresponds with the distribution of Geometric Excess Return rates along the Selection Ratio variable, i.e. The inverse-U relationship between portfolio variance and Selection Ratios can similarly be seen for excess return rates.

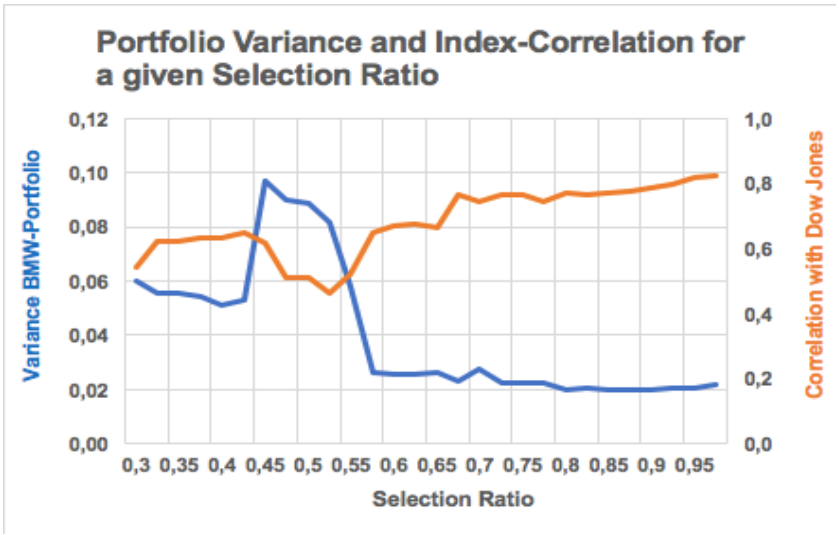


Figure 10: Portfolio Variance and Index-Correlation for a given Selection Ratio

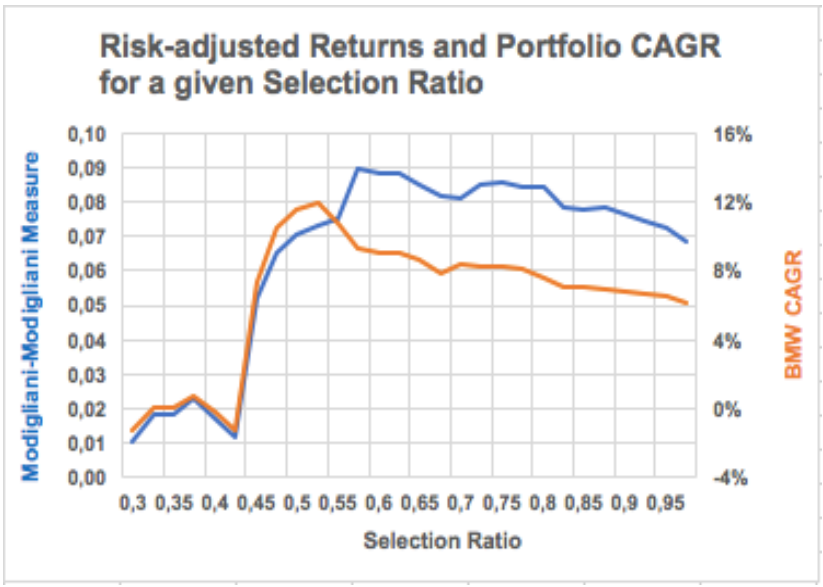


Figure 11: Risk- adjusted Returns and Portfolio CAGR

Figure 11 displays the M2 measure and our portfolio CAGR based upon different Selection Ratios. The figure gives further confirmation that a Selection Ratio in the range of 0.5-0.6 seems to be beneficial for our method in terms of absolute return, but also risk-adjusted returns.

The results presented in figure 4 (p. 12) illustrate the detailed risk-adjusted returns for different Selection Ratios based on the Ex post Sharp Ratio. We notice that the Ex post Sharp Ratio is above 0 for the whole interval between 0.45-0.65. We also notice that the peak of the Ex post Sharp Ratio, at 0.2296, comes when 0.525 is selected as buying criteria. One can argue that our strategy tends to outperform the DJIA on a risk-adjusted basis within certain criteria. However, the great difference between the number of stocks held in our portfolio and the DJIA makes the measure more unstable, and one can argue that it doesn't produce greater insight to our risk-adjusted return. We can, however, say that for the greater part of the interval of Selection Ratios the portfolio tends to be close to the DJIA on a risk-adjusted basis.

The opportunity cost

When evaluating the performance of a specific investment portfolio, it is fundamental not only to analyze the "returns" part, but also to consider the "losing" part, or better, to consider the opportunity cost of not investing the money in other stocks or portfolios.
In particular, we will keep our focus on the DJIA, but we will consider the stocks which were not included in the investment portfolio because laying out of the stock Selection Ratio criteria.

The BMW-method focus on stocks that are performing extremely well or extremely bad, predicting that, in the long run, the pattern will correct again towards the Average CAGR. So, what about stocks with less volatility in their stock prices, but that increased following a pattern during the 20-years period?

By setting the Selection Ratio at 0.525, we are keeping out of our portfolio some very important and historically-well performing stocks like: Honeywell, American Express, IBM, Mc Donald's, Boeing, Coca Cola, P&G, Intel, etc.

OPPORTUNITY COST PORTFOLIO			
	%	CUM RETURN	VAR
American Express	12.50%	393.38%	0.157
Honeywell	12.50%	348.44%	0.059046
IBM	12.50%	438.25%	0.079147
MacDonald	12.50%	536.50%	0.061771
Boeing	12.50%	292.43%	0.088055
Coca Cola	12.50%	157.58%	0.025236
P&G	12.50%	312.50%	0.024296
Intel	12.50%	221.52%	0.148315
PORTFOLIO TOTAL	100.00%	337.58%	0.08033

Figure 12: Opportunity Cost Portfolio

In figure 12, one can observe a rough estimation of what an "opportunity cost portfolio" would look like. This portfolio includes 8 firms that have never been included in the portfolio because their Current CAGR never fell under the Average CAGR, or fell for a Selection Ratio lower than the one selected. They all have the same weight of 1/8 and, for each of them, we calculated the Cumulative Return over the 20 years period and the variance of their returns over the investing period. By doing so, we will have an idea of both the returns and the risk.

Comparing our portfolio, using the best-performing ratio of 0.525 and the opportunity cost portfolio, it is easy to notice that the aggregated returns are much higher in our portfolio (853.7 % compared to 337.58 %), while the variance is close to identical (0.08). This result strongly supports our portfolio due to the slightly higher diversification in the opportunity portfolio with a higher number of stocks included (8 compared to an average of 7.3 of our portfolio). What we can conclude from this short analysis is that, for Selection Ratios ranging from 0.3 to 0.45 and from 0.8 to 0.975, our portfolio is poorly performing and is beaten both by the index and by the opportunity cost portfolio. For ratios from 0.45 to 0.775, instead, our portfolio is outperforming the opportunity cost portfolio, generating higher returns with similar risk. Therefore, one can argue that our portfolio is the most preferable compared to the opportunity cost portfolio.

Disadvantages of the BMW-Method

Interpreting the measure of the Average CAGR is challenging, because it requires computing the arithmetic mean of a concept that is based on a geometric mean (the Current CAGR).

This results in a form of complexity that can be difficult to understand: the respective Current CAGRs used to build the Average CAGR allows for compounding effects. For example, a stock's price that rises by 10% a year for two years will result in an overall price increase of 21% (higher than just the sum of its growth rates). If the Current CAGR is positive, the current stock price will always be higher than the base year's stock price.

Computing the Average CAGR will then cancel part of this effect. That results in the oddity that a stock can have an Average CAGR less than zero, but still trade at a higher price than the base year's price. In this case, the BMW-method assumes a stock with a negative long-term growth tendency unworthy of investment, as it claims that a stock's Current CAGR will move towards the (long-term) Average CAGR.

Let's take a simple example to illustrate this: We imagine a stock which price falls by 50% in the first year and rises by 100% in the second year. After two years, the stock has the same price as in the base year (0.5 x 2.0 = 1.0), trading at a Current CAGR of 0%. The Average CAGR would be 25%, though: 0.5 x (-50% + 100%). The BMW-method now suggests investing in this stock, which to some people, may seem counter-intuitive, since most of us would prefer to evaluate investments in the terms of a concept allowing for compounding interests. Therefore, one can argue that the BMW-method is so comprehensive to understand that the method has a disadvantage compared to simpler and straightforward investment strategies.

At last, we acknowledge that our virtual portfolio solely consists of firms defined as "mega caps" with high credit ratings, meaning the chance for bankruptcy is low. However, using the BMW-method on small caps could cause issues since they are more likely to have financial troubles of the scale that causes bankruptcy. Also, one can argue that the CAGR is unusable for small caps since it´s even less likely that they will have a constant and steady growth rate. Both observations are important, because a fundamental aspect of our model is that firms "survive" bear markets and can rebounce back to growth. This finding causes us to think that the BMW-method may only work for a certain type of company and that it actually is something particular for the DJIA, which directly contradicts Klein´s hypothesis "*not something particular for the DJIA*".

Conclusion

This term paper examines an investment strategy termed as BMW-method, following an approach that examines a stock's Compounded Annual Growth Rate. We made a model in Microsoft Excel that allowed us to compute a number of return and excess return measures for a variety of Selection Ratios. Furthermore, our model provides several statistical risk measures alongside to help us understand how the BMW-method portfolio affects the relationship between risk and return in comparison to the DJIA.

Using the respective constituents of the DJIA as investment universe and a dataset of yearly stock price information for 31 years, we could show that our portfolio did not consistently outperform the Dow Jones. While it does outperform the benchmark for some of the suggested Selection Ratios, several risk measurement statistics indicate that these excess returns come at the price of substantially higher risk. We also note that the method might only work for a mega caps and not small caps. However, we concluded that as the BMW-method portfolio size was consistently small, interpreting these findings is very challenging, and definitely not conclusive.

Looking back at the time we have spent working on this term paper, we have hugely enjoyed the challenge to work on a topic we could choose by ourselves. Attempting to develop analytical tools and instruments to examine an investment approach we had read about on the internet and finding out how to apply this approach to a historical real-world investment universe was a truly enlightening experience. Nonetheless, we are very aware of the enormous limitations of our findings and would enjoy working on it again in the future, only with greater knowledge and wisdom at hand.

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