

Pairs Trading in the U.S. Defense Industry

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Executive Summary

Since its inception in the 1980s, pairs trading has been a popular market-neutral trading strategy. It involves finding a stock pair whose prices move together, and opening and closing opposing positions as the prices oscillate to and from their equilibrium spread. However, the strategy has shown signs of declining profitability as the years have passed.

This paper provides an analysis and evaluation of the prospective profitability of pairs trading in the U.S. defense industry. Working from the thesis question *Is pairs trading still a viable strategy to earn high risk adjusted returns?*, we have sought to discover whether potential profits from pairs trading justify the risk involved with such a strategy.

Our method of pair selection combined heuristic selection, calculation of correlation and beta, and testing for cointegration in stock prices in the period 01.01.2000-31.12.2009. This process resulted in the selection of two large-cap U.S. defense contractors, Lockheed Martin (LMT) and Raytheon (RTN), as our trading pair. To develop a trading strategy while avoiding data snooping, we backtested on stock prices between 2000 and end-of-year 2009. Our method included normalizing the spread with a moving average, and optimizing the strike value for opening and closing positions. We selected to trade on a sum of $10,000 for each trade, not reinvesting the profits from each trade. To implement this strategy, we developed a model in Excel. The model accounted for transaction costs and alternative cost.

We implemented our strategy using RTN and LMT stock prices spanning the period 2010-2017. This resulted in a $8,288.81 profit for the whole period. The resulting portfolio standard deviation of 0.5386% gave us a Sharpe ratio of 1.015, indicating that our risk-adjusted payoff was satisfying. Our net return for the period was 82.88%, which constitutes an 11.84% average, yearly return. Our returns were not compounded due to not reinvesting profits. The overall results indicate that pairs trading still is a viable strategy to earn high risk adjusted returns.

The paper also investigates the fact that our analysis has some limitations and weaknesses. Some of the limitations include: the assumption that trades can be executed immediately and to closing prices, without having to pay the bid-ask spread; the sensitivity to changes in trading costs; the assumption that non-integer volumes of stocks can be traded; no collateral requirement when shorting stocks; and a thesis question fallibility.

1. Introduction

In this paper we will look at a market neutral strategy called pairs trading. This strategy was first implemented through a project organized by Morgan Stanley & Co. in the mid-1980s. (Gatev, Goetzmann, & Rouwenhorst, 2006). The purpose was to replace the investor's skills and investment psychology with disciplined investment rules. The strategy was very successful for the team and generated a surplus of $50 million in 1987. Since then the strategy has developed further, but its success has declined over the years. Gatev, Goetzmann and Rouwenhorst (2006) claims that "one view of the lower profitability of pairs trading in recent years is that returns are competed away by increased hedge fund activity", while Do, Faff and Hamza (2006) suggest hedge fund activity plays a minor role compared to increased arbitrage risk such as synchronization risk, noise trader risk, and fundamental risk. The purpose of this paper is to uncover whether pairs trading still is a viable road to profits, despite its declining profitability trend.

Our thesis question is: *Is pairs trading still a viable strategy to earn high risk adjusted returns?*

To answer this, we will select a suitable asset pair and analyze this to develop and implement a pairs trading strategy and model. We start off by presenting the theoretical framework supporting pairs trading. Thereafter, we select a suitable pair of stocks based on their correlation, beta and cointegration. Then we develop a strategy based on backtesting historical data. Furthermore, the model is constructed, before we implement the strategy and discuss the results. Lastly, we look at constraints, limitations and weaknesses in our paper before we conclude on our thesis question.

2. Theory

Pairs trading denotes a set of market neutral trading strategies in which portfolios are constructed from just two securities in an inefficient market (Vidyamurthy, 2004). The fundamental idea behind pairs trading is that market inefficiencies cause securities to be mispriced relative to each other, such that one security becomes relatively cheap and the other relatively expensive (Harris, 2002). A portfolio devoid of market risk can thereby be constructed by opening and closing positions subject to the spread. When the spread deviates from its mean, positions in the two securities are opened. When the spread reverts to its mean, the positions are closed. In this paper we will discuss a specific pairs trading strategy known as *statistical arbitrage pairs trading*. This strategy assumes that securities with similar characteristics must have prices that move together, and thereby an equilibrium spread. By finding securities whose prices have been historically correlated, one can trade profitably in these two securities when the spread widens and deviates from its historic mean (Vidyamurthy, 2004). The idea is to go long in the underpriced asset and short in the overvalued one, betting on the spread to converge back to its historic mean. This means buying the stock whose price has decreased relatively, and shorting the stock whose price has increased relatively. Once the spread converges back to its equilibrium mean, the positions are closed and a profit is made. Since the two assets are correlated and subsequently have similar betas, the opposing short- and long positions negate each other, making statistical arbitrage pairs trading a market neutral strategy.

This strategy is profitable and possible due to inconsistencies in the pricing of the two securities' common fundamental valuation factors, or because traders misprice instrument-specific factors (Harris, 2002). However, pairs trading can backfire when changes in relative prices are due to changes in instrument-specific factors, which may cause permanent price movements which are detrimental to a pairs trader's position.

The concept of market neutrality merits further explanation, since this is a major motivation for pairs trading. Market-neutral strategies seek to improve overall returns while defusing market risk and volatility (Alliance Bernstein, 2012). By combining certain long and short stock positions in a portfolio, a neutral strategy can be formed to avoid market movements, and returns made from pairs trading can be uncorrelated with returns in the market. The preferred measurement for a stock's market risk is the Beta (β_i), which is used to measure the sensitivity to market movement. A statistician would define beta of a stock *i* as:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \tag{2.1}$$

Where:

β_i =The beta of stock i	σ_{im} =covariance between the stock return and the market return	σ_m^2 =variance of the returns on the market

Table 1: Beta description

If a stock has $\beta > 1.0$, it tends to amplify the overall movements of the market, and can be categorized as a high-volatility stock. Whereas stocks with $0 \leqslant \beta \leqslant 1.0$ have low volatility and moves less than the market. Stocks with a negative beta will move in the opposite direction of the market, indicating a negative correlation. Since the market is the portfolio of all stocks, the average stock has $\beta = 1.0$

Subsequently, a market neutral strategy involves creating a portfolio with a beta of 0, thereby eliminating systematic risk. To form such a portfolio one could find two stocks with opposite beta values, or go long and short in two stocks who are highly correlated with similar beta values. The latter is preferred, since few stocks have a negative beta-value (Vidyamurthy, 2004). This creates a portfolio with a beta-value of zero, entailing zero market risk.

According to Vidyamurthy (2004) a long-short position with $\beta = 0$ gives zero market return, and profit or loss from such strategies can only be explained by the residual component or residual return on the portfolio, θ_i. In essence, a market neutral trader can now focus on forecasting and trading the residual returns. According to Vidyamurthy (2004) the mean on the residual return is zero, and he states that it is reasonable to expect a strong mean-reverting behavior of the residual time series. This behavior can be utilized to predict future returns.

3. Data and premises

To conduct our analysis and implement our pairs trading strategy, we created a scenario where today's date was 01.01.2010. By backtesting on data spanning the period 01.01.2000 to 31.12.2009 (2000-2010), we developed a pairs trading strategy and model to implement from 01.01.2010 until 31.12.2016 (2010-2017). We adopted this procedure to ensure a realistic scenario where we had no prior knowledge of future developments, thereby avoiding "data-snooping", which refers to the danger of fitting the data to a particular model and then claiming that the model is indeed very profitable (Reverre, 2001). The data on which we based our analysis was gathered from Bloomberg, CRSP through WRDS, Yahoo finance and DataStream.

3.1 Selecting pairs

The essential criteria for pair selection is that the stocks' prices must have moved together historically (Gatev, Goetzmann, & Rouwenhorst, 2006), thereby indicating that they might continue to move together in the future as well. In this paper, we have relied on a four-step process to identify and select an asset pair. We started off by implementing a heuristic technique to group stocks along common identifying factors and influences, thereby narrowing our selection. Thereafter, we determined the correlation between the stocks' prices, before looking at beta values of the highest correlated pairs. Finally, we selected our preferred candidate pair and tested whether this asset pair was cointegrated and suitable for pairs trading.

3.2 Heuristic selection

Instead of immediately performing quantitative analysis on the whole asset universe, we relied on a heuristic technique to narrow down our asset population to selections which were likely to include stocks whose prices had moved together historically. Considering our data sources, we first decided to limit our population to US stocks traded on either the NYSE or NASDAQ exchanges. Thereafter, we proposed to segment stocks according to their industry, and to further analyze those industries in which we believed companies faced the same market risk and had strong common factors influencing stock prices. One such industry is the American defense industry. In addition to being influenced by common factors such as commodity prices and interest rates, we believed that American defense contractors' stock prices would largely be dependent on the US defense budget, since this is by far the largest military budget in the world (Sipri, 2017) and the main driver of demand for American military equipment (Bloomberg, 2017). Furthermore, by limiting our selection to include only large-cap companies with a wide array of products, we reduced the volatility of the stock prices we were analyzing, as individual contract wins and sales have less of

an impact on stock prices of large-caps than for smaller companies. As such, we ended up with a selection of six companies: Boeing (BA), Raytheon (RTN), Northrop Grumman (NOC), General Dynamics (GD), L3 Technologies (LLL) and Lockheed Martin (LMT). The companies' stock prices in our backtesting period are shown in Figure 1.

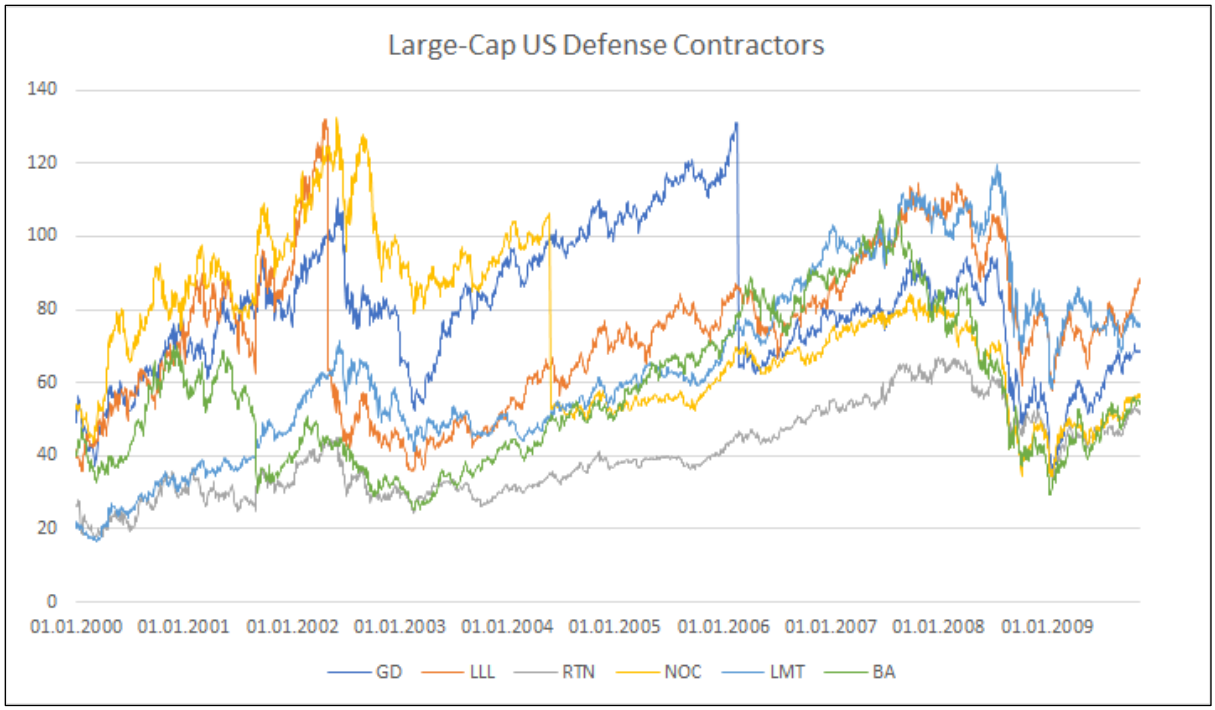


Figure 1: Stock Prices of Large-Cap U.S. Defense Contractors (2000-2010)

3.3 Correlation and Beta

Having narrowed our selection, we subsequently calculated the correlation coefficient between each pair of stocks in the period 01.01.2000 to 31.12.2009. The correlation coefficient ranges between 1 and -1, where a coefficient of 1 indicates complete positive correlation and -1 indicates the opposite. In other words, a positive correlation means that the prices of the two stocks will increase and decrease simultaneously, while negative correlation means that stock prices move inversely of each other. Table 2 shows the correlation matrix for our selection. It is immediately apparent that the stock prices of Raytheon (RTN) and Lockheed Martin (LMT) are highly correlated, having a coefficient close to 1, subsequently indicating that these two companies could constitute a suitable candidate pair.

On an interesting side note, some of the poor correlation between GD, NOC and LLL can be explained by the fact that they all had a 2:1 split in the period we ran the correlation tests, giving the shareholders two shares for every share owned pre-split (Stock Split History, 2017). These splits can be seen in the sudden extreme drops in Figure 1. Further investigation on the different

correlations was not necessary since we were satisfied with RTN and LMT. However, if these two had turned out to be unsuitable, we could have calculated the "true" correlation by reversing the share emissions.

	GD	LLL	RTN	NOC	LMT	BA
GD	1					
LLL	0.25769783	1				
RTN	0.13977819	0.74810658	1			
NOC	0.15408609	-0.0308271	-0.2256476	1		
LMT	0.15071065	0.64820674	**0.95978915**	-0.2011215	1	
BA	0.24059265	0.65322904	0.71370754	-0.1891852	0.65381564	1

Table 2: Correlation Matrix

Furthermore, we extracted each stock's beta value for our backtesting period to analyse their exposure to the overall market, represented by the S&P 500 index. Table 3 shows each stock's beta for the same period as above, calculated in Excel by using formula 2.1. Raytheon and Lockheed Martin's beta values are highlighted and are clearly very similar. By using formula 3.1 we can calculate the portfolio's overall beta value. This, of course, depends on which stock you acquire a short or long position in, and in our case, we will get two different beta values as our positions are always weighted equally. If we go short in RTN and long in LMT, the portfolio beta will be -0.015 and if we do the opposite the beta will be 0.015. These values are close to 0, which is one of the requirements to create a market neutral portfolio.

$$\beta_p = w_1\beta_1 + w_2\beta_2 + \cdots + w_n\beta_n \tag{3.1}$$

	BETA	CORR S&P 500
GD	0.638	0.120
LLL	0.661	0.434
RTN	**0.549**	**0.295**
NOC	0.540	-0.082
LMT	**0.534**	**0.179**
BA	0.901	0.745

Table 3: Calculated Beta Values

3.4 Cointegration

Lockheed Martin and Raytheon's correlation coefficient and beta values indicated that these two were a suitable trading pair. However, to strengthen our analysis of the pair's tradability, we conducted a cointegration test on their stock prices. Cointegration entails that a linear combination of two non-stationary time series itself can be stationary. By conducting a cointegration test, we discover whether the two stocks' price time series are first-order integrated and non-stationary, and in that case, whether the two stocks are cointegrated. Since such time series can only be cointegrated if there is a genuine relationship between the two, the existence of a cointegration relationship eliminates the possibility of spurious correlation, and implies that the two stocks share a long-term equilibrium relationship (Hucka & Afawubob, 2015). This long-term equilibrium relationship is represented through the price spread between the two. Furthermore, if the stocks' time series are cointegrated, the portfolio will be mean reverting such that the spread will always revert to its long-term equilibrium value, so that any deviations from this equilibrium are only temporary fluctuations (Puspaningrum, 2012). Since pairs trading depends on asset mispricing and mean reversion, confirming a cointegration relationship between the two stocks confirms that mean reversion, the most essential foundation for pairs trading, exists. Cointegration does not guarantee that mean reversion will take place in the future, since analysis can only describe past events, but the existence of such a relationship gives a strong indication that mean reversion is likely (Puspaningrum, 2012).

To uncover whether a cointegration relationship exists between Raytheon and Lockheed Martin, we conducted an Engle-Granger test using gretl data analysis software. The results of this test are given in the appendix. Dicky-Fuller tests (as part of the Engle-Granger test) of each stock for non-stationarity with a unit root resulted in the null hypothesis for both stocks not being rejected, such that there was not enough statistical evidence to infer that these were stationary. Furthermore, the unit-root null hypothesis for the residuals of the cointegration regression was rejected. As such, our test indicated that the Raytheon-Lockheed Martin pair was cointegrated, which in addition to being highly correlated and having ideal betas, made them very suitable for pairs trading (Hucka & Afawubob, 2015). We therefore selected these two stocks as our asset pair for further analysis. A brief description of the companies is presented below.

3.5 Company descriptions

Lockheed Martin

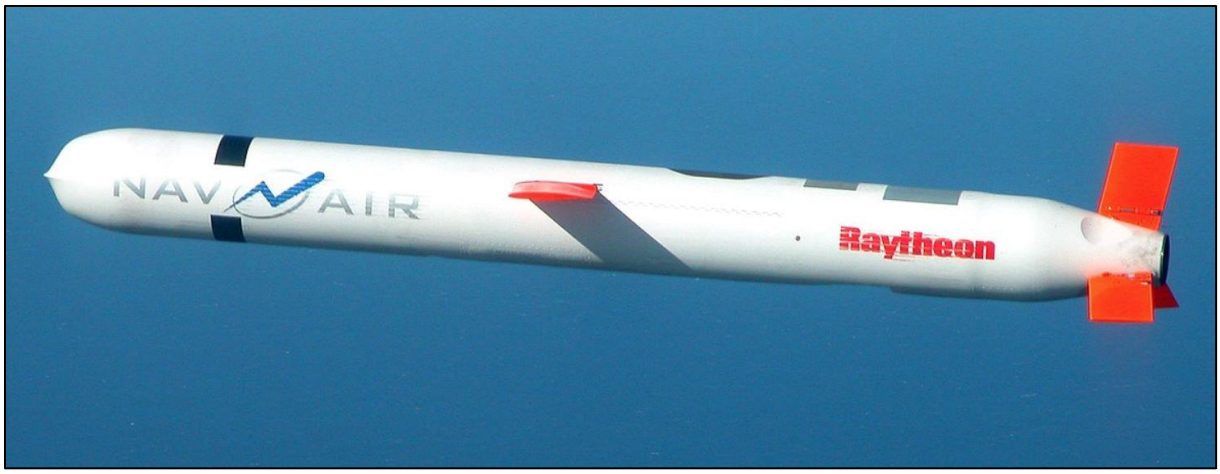
Lockheed Martin (LMT) is an aerospace and defense company specializing in space, telecommunications, electronics, information and services, aeronautics, energy and systems integration. The company is a major supplier of goods and services to the U.S. government, in particular to the department of defense, while simultaneously presenting a strong international presence. Notable LMT innovations include the F35 fighter jet, the U-2 spy plane, the Trident-series ICBMs and the AEGIS Ballistic Missile Defense System (Bloomberg, 2017).



Picture 1: Lockheed Martin's SR-71 Blackbird (Lockheed Martin, 2017)

Raytheon

Raytheon (RTN) specializes in military goods and services for reconnaissance, targeting, navigation, missile systems, unmanned systems, radars, satellite communications, intelligence and cybersecurity. The company's main customer is the U.S. government, which accounts for roughly 70% of its revenues. However, RTN also has a major presence as an international defense contractor, and is renowned for the development of missile systems such as the Patriot-, Sidewinder-, and Tomahawk missiles (Bloomberg, 2017).



Picture 2: A Raytheon Tomahawk Cruise Missile (Raytheon Company, 2014)

3.6 Pair analysis

A short analysis of why exactly LMT and RTN have such a high correlation coefficient is both interesting and adds valuable insight into why these companies might exhibit mean reverting spread behaviors. To add depth to our analysis, Table 4 exhibits a selection of key financial and strategic metrics for both companies.

Metric	LMT	RTN
Market Cap	$90.387 Billion	$53.475 Billion
Revenue (2016)	$47.248 Billion	$24.069 Billion
P/E	$24.8	$24.6
Revenue by End User	• 70.8% U.S. • 26.9% International. • 2.30% Commercial & Other	• 68.6% U.S. • 31.4% International
Revenue Breakdown	• 42.48% Aerospace & Defense • 19.91% Missile & Space • 37.61% Military Aircraft	• 45.77% Aerospace & Defense • 27.72% Missile & Space • 26.50% Network Integration

Table 4: LMT vs RTN: Key Figures (Bloomberg, 2017)

As we see from the table, LMT is a much larger company than RTN in terms of market capitalization and revenue. However, looking at other metrics and our overview of the companies, we see that there are several similarities which go a long way to explain why their prices correlate. First, both companies have similar revenue exposure in terms of customers and geography. Roughly 70% of revenue stems from sales to the U.S. government (Bloomberg, 2017). Such a dynamic is likely to make these companies' performance heavily dependent on U.S. government expenditure, which in turn make them dependent on the federal defense budget. If the budget increases, sales (and consequently stock prices) for these two companies are likely to increase, and vice versa. Furthermore, both companies specialize in delivering many of the same military and defense technologies. The similarity of their business model is illustrated by each company's revenue breakdown. Both derive roughly 40-45% of their total revenue from aerospace and defense products, and similarly have large missile and space divisions. Consequently, this qualitative analysis also points in the direction of high correlation and comovement in LMT's and RTN's stock prices.

4. Strategy and Model

To answer our thesis question, we need to develop a pairs trading strategy and build a model where we can implement our approach. This involves calculating a suitable metric for deviations from the equilibrium spread and determining strike values for opening and closing positions. We also need to account for external parameters such as trading costs (TC) and alternative cost, and select measures with which to evaluate our performance.

4.1 Performance measures

To evaluate the results and optimize our trading strategy, we incorporated five performance measures in our model: Sharpe Ratio, Fama-French five factor model, Return on Investment (ROI), Profit and Loss (P&L), and Excess Returns. The formulas for each performance measure are listed in Table 6, apart from the Fama-French five factor model, which is explained in detail in a separate section below.

Sharpe Ratio

The Sharpe Ratio, originally known as the *reward-to-variability ratio* (Sharpe, 1994), is the main performance indicator used in this paper. This metric shows the excess return as a function of standard deviation (Brealey, Myers, & Allen, 2017), and measures how well the portfolio's return compensates for its risk. In this sense, it is a measure of risk-adjusted performance, which fits well

with regards to our thesis question. In this paper we have adjusted the formula for the Sharpe ratio to correspond with the 252 U.S. trading days in a year (NYSE, 2017).

Fama-French five factor model

The Fama-French three factor (FF3F) model uses three factors to determine expected return. This is a model that is widely used to measure performance of mutual funds, pension funds and other professionally managed portfolios, and as such, is a useful indicator of our strategy's performance (Brealey, Myers, & Allen, 2017). The FF3F model takes into account a market factor (1) from the CAPM model, a size factor (2), and a book-to-market factor (3). The factors are measured respectively by the return on the market index less the risk-free interest rate; the return on small-firm stocks less the return on large-firm stocks; and the return on high book-to-market-ratio stocks less the return on low book-to-market-ratio stocks. In recent years the model has been supplemented by another two factors: profitability (4) and investment (5). The former is measured by the difference between the returns of firms with robust (high) and weak (low) operating profitability, and the latter is measured by the difference between the returns of firms that invest conservatively and firms that invest aggressively (Fama & French, 2014). The FF5F model uses the following formula to calculate expected returns:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} \quad (4.1)$$

Where:

R_{it} is the return on security or portfolio *i* for period *t*.

R_{Ft} is the risk free return.

R_{Mt} is the return on the value-weight (VW) market portfolio.

SMB_t is the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks.

HML_t is the difference between the returns on diversified portfolios of high and low book-to-market-ratio stocks.

RMW_t is the difference between the returns on diversified portfolios of stocks with robust and weak profitability.

CMA_t is the difference between the returns on diversified portfolios of low and high investment stocks, which we call conservative and aggressive.

e_{it} is a zero-mean residual.

b_i, s_i, h_i, r_i, c_i are coefficients to the portfolios return and if these capture all variation in expected returns, the intercept a_i will be zero for all portfolios or securities *i* (French, 2017).

The FF5F model shows us the cost of equity (COE) of an investment, which indicates what level of return the investment must make in order for it to be worth the risks it is exposed to. Subsequently, we want to see a COE on our portfolio that is lower than the return of our pairs trading portfolio.

Profit and Loss

In our model we calculated the total P&L after each closed position and added it up to track our total profit from day to day throughout the period. This was done both with and without transaction costs (TC).

Return on Investment

We calculate the strategy's return on investment to easily compare it with other investment opportunities. In this paper we have assumed that profits are not reinvested and that losses are not subtracted from the investment value. Subsequently, the cumulative ROI does not represent a compound return.

Excess Return

We also calculate the strategy's excess return, which shows the return on investment which exceeds the risk-free rate.

4.2 Setting up the trading model

To implement our strategy, we first selected a suitable metric for measuring the spread, before constructing a dynamic model in Excel, which was based on the basic dynamics of pairs trading. Thereafter, we inserted our external parameters before backtesting on our asset price data in the period 2000-2010 to develop an optimal trading strategy (strike values, moving average) for our future period (2010-2017).

The basic strategy and model design of pairs trading is simple: open a position when stock prices move in a way which deviates from the long-term equilibrium spread, and close the position when the spread reverts to its mean value. In our model, we decided to follow an approach in which the spread, symbolized as Δ, is normalized by its standard deviation in accordance with a moving average:

$$\Delta_{norm} = \frac{\Delta - MA_{t\,(optimized)}(\Delta)}{\sigma_{t\,(Optimized)}(\Delta)} \quad (4.2)$$

This formula for normalization is based on Reverre's (2001) suggested formula for normalizing the difference between a stock price and its ratio value. By normalizing the spread in such a manner, we produce a relative metric based on standard deviations instead of absolute values, in which random variability has been reduced through the incorporation of a moving average. This is because moving average is a tool to forecast time series and remove random variation in statistics. There are two types of moving average: the simple moving average (SMA) and exponential moving average (EMA). We have chosen the SMA for our model as it is more stable and doesn't weight recent price changes as heavily as the EMA (Keller, 2014). EMA might force us to close or open a position when it's not favorable, we avoid this by choosing the SMA.

Subsequently, we can formulate a trading strategy and develop a model where the opening and closing strike price is based on the normalized spread. Given the fact that the spread is normally distributed, the advantage of normalizing such metrics is that we know that the normalized spread is distributed with expected mean 0 and standard deviation 1 (Reverre, 2001). This means that we can easily identify the probability of the occurrence of different spread values. As we see from Figure 2, the spread's distribution is not fully normally distributed, but we have enough observations to assume normality in accordance with the central limit theorem (Keller, 2014). The distribution of the normalized spread can be seen in Figure 3.

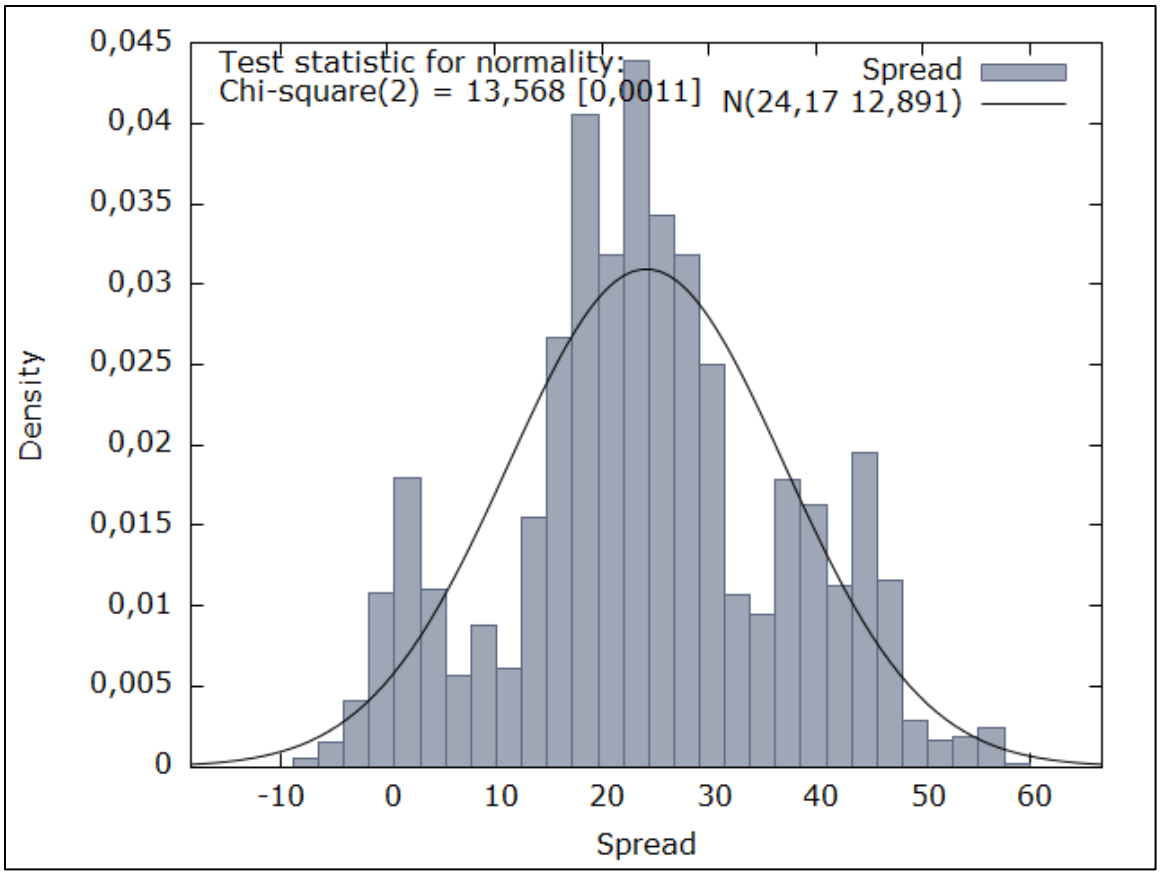


Figure 2: Distribution of Spread

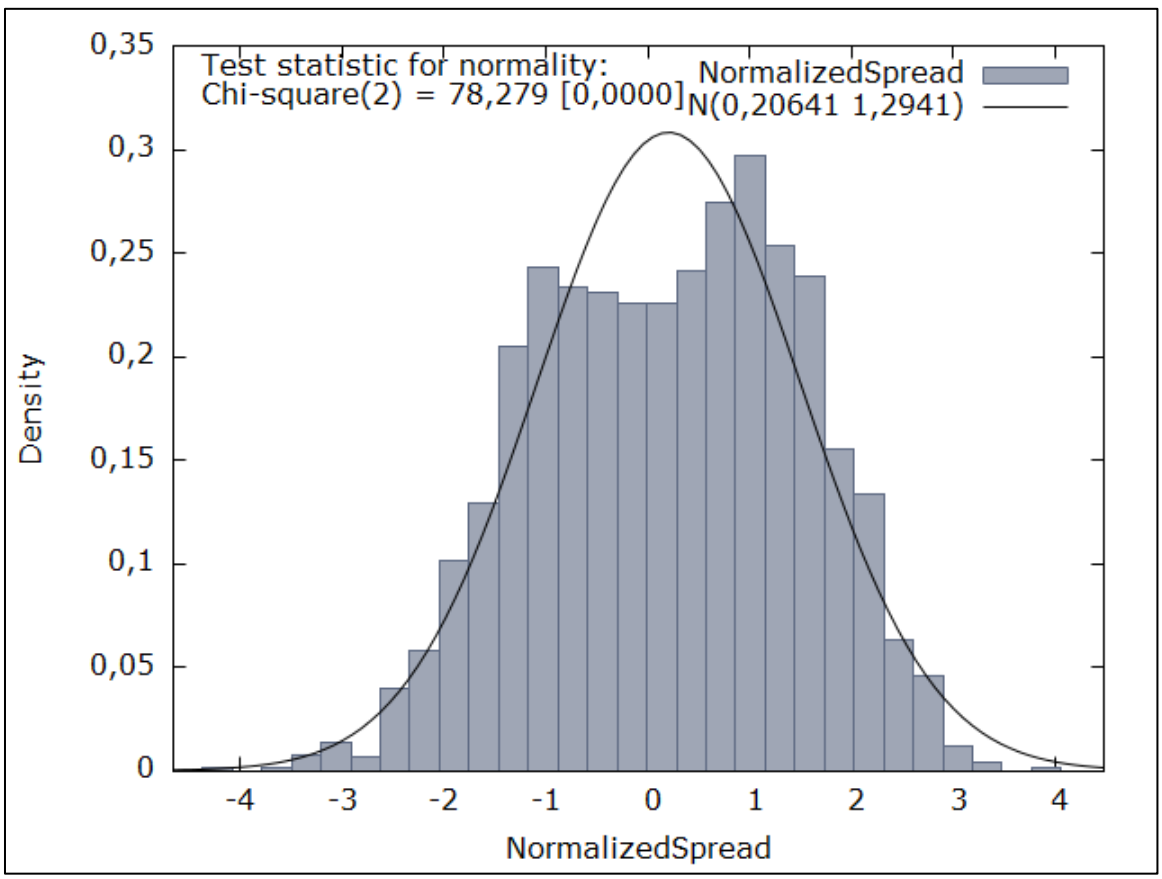


Figure 3: Distribution of Normalized Spread

4.3 Optimizing the strike value and moving average

Our strike values determine when to take long/short positions and when to close them, while the moving average determines the degree to which random variability is reduced. In order to determine a strike value and moving average, we used the goal-seek function in Excel while backtesting data from the period 2000-2010, using Sharpe ratio as a performance indicator. In a perfect model we would optimize the strike value and the moving average simultaneously. Though we tried this, it turned out to be too complex adding another dimension, and outside our range of ability. Therefore, we set the moving average to 30 days at first as a starting position, as suggested by Reverre (2001). We then optimized the strike value by maximizing the Sharpe ratio while changing different strike values, and holding everything else constant. The highest Sharpe ratio of 1.4138 was achieved at a strike value of 0.20584. Since our normalized spread is calculated on the basis of standard deviations, $\Delta=0$ indicates the long-term equilibrium value of the spread. We subsequently modified our model to close positions when the normalized spread equaled zero, indicating that the price difference had returned to its mean value.

We also decided to do a sensitivity analysis to compare the different strike values we could have chosen. Figure 4 proves that our chosen strike value of 0.20584 results in the highest Sharpe ratio. We see that a strike value of 1.05 gives larger profits than 0.20584, but as mentioned we chose to optimize the Sharpe ratio because this measurement is risk adjusted. With the strike value of 0.20584 as a constant, we optimized the moving average, and found that 30 days is the optimal value for our dataset, which is, remarkably, the same amount of days as Reverre (2001) suggests.

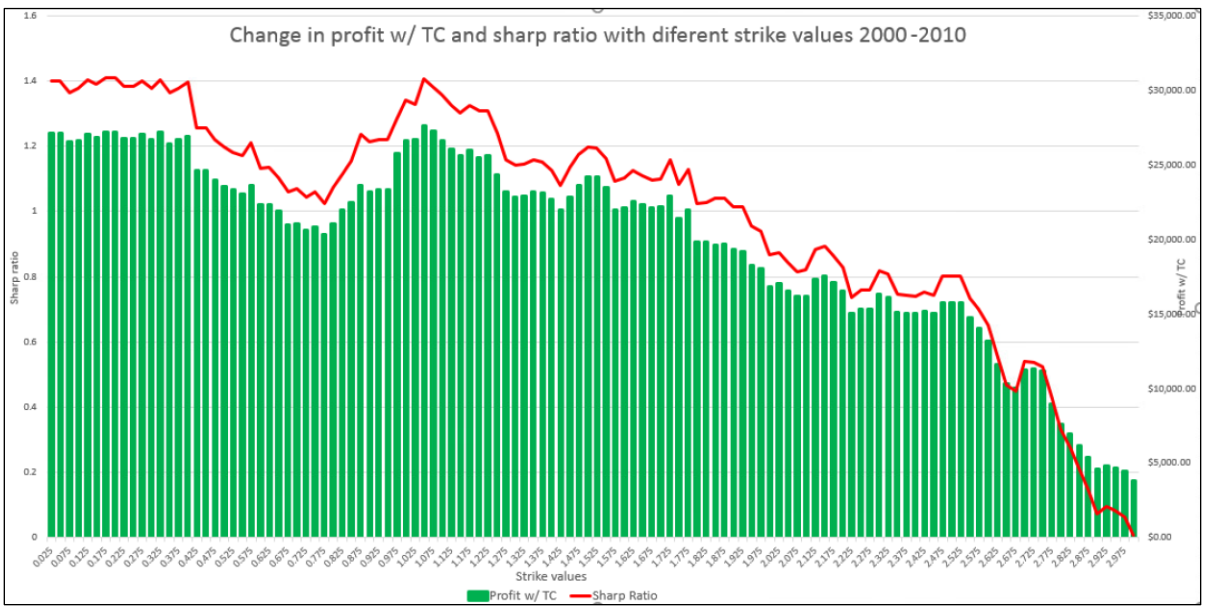


Figure 4: Sharpe Ratio and P&L with different strike values (2000-2010)

4.4 Accounting for Transaction and Alternative Costs

TC and alternative costs (AC) have a major impact on any trading strategy and can determine whether a strategy is profitable or not. They must therefore be accounted for in our model.

Transaction costs

When trading securities, a trader incurs TC such as commissions, fees, and the bid-ask spread. We have assumed that purchasing and selling assets is done at the close price, such that the spread transaction cost is eliminated. However, other TC remain. In pairs trading, the simultaneous opening or closing of a long- and short position means TC is incurred twice: once for the short position and once for the long position. The TC we incur are brokerage fees from buying and selling stocks and stock loan fees from borrowing stocks for the short position. Brokerage fees are incurred on a one-off basis for each trade, while stock loan fees are interest payments which accrue for each day the stock is held.

We used Interactive Brokers' fees to find the appropriate TC for our model. This broker offers a $0.005 fee per stock traded (InteractiveBrokers, 2017a). If we calculate this as a percentage the brokerage fee would be 0.0046%. Our strategy calculates this fee four times for each pairs trade: twice when opening and twice when closing. The loan fee varies from stock to stock, and is determined by how easy it is to borrow the specific asset**.** Many brokers operate with an "easy to borrow list" which shows the shorting availability for different companies. LMT and RTN are both on the "easy to borrow list" (mbtrading, 2017). For our "easy to borrow" classification, Interactive Brokers offers 0.25% in stock loan fee, calculated annually, which also accounts for weekends and holidays (InteractiveBrokers, Calculating the Cost of Borrowing Stock at Interactive Brokers, 2017b).

Alternative cost

To see if our $10,000 could have been invested better, we have compared our strategy with an alternative cost. This AC is represented by the 10-year US treasury bond from 2010, which we assume to provide a risk-free annual return of 3.83% (CRSP, 2017). The model's TC and AC are summarized in Table 5.

4.5 The optimized strategy and model

In this section we have summarized our strategy and the parameters and performance measures for our model, apart from the FF5F model, which is presented in the appendix.

Parameters

Costs	Amount	Extra information
Brokerage fee	$0.005 per stock or 0.0046%	From Interactive Brokers Calculated
Stock loan fee	0.25%	From Interactive Brokers
Alternative cost	3.83%	From US government bonds in 2010

Table 5: Parameters

Performance Measures

Performance measurements	Formula	Extra information
Profit and loss (π)	$\pi = Gains - Losses$	
Return on investment (ROI)	$ROI = \frac{Net\ income}{Investnment}$	Assuming investment = $10,000
Sharpe ratio (S_p)	$S_p = \frac{((1+\bar{r_p})^{252}-1)-r_f}{\sigma_p * \sqrt{252}}$ Where: $\bar{r_p}$ =Average of daily percentage return on the portfolio r_f =Risk free rate σ_p =standard deviation on the portfolio	Assuming trading days = 252 Assuming risk free rate = 0.0383
Excess return (Er)	$Er = \%return - risk\ free\ rate$	Assuming risk free rate = 0.0383

Table 6: Performance Measures

Strategy

Our normalized metric for the spread, such that deviations from the long-term equilibrium are easily measured:

$$\Delta_{norm} = \frac{\Delta - MA_{t\ (optimized)}(\Delta)}{\sigma_{t\ (Optimized)}(\Delta)} \tag{3.1}$$

Since backtesting gave t=30 days as the optimized period:

$$\Delta_{norm} = \frac{\Delta - MA_{30}(\Delta)}{\sigma_{30}(\Delta)} \quad (3.2)$$

On the basis of the above metric, we have selected strike prices so that we open and close positions according to the following:

Open position when: $\Delta_{norm} > |0.20584|$

Close position when: $\Delta_{norm} = 0$

Short RTN, long LMT when: $\Delta_{norm} < 0.20584$

Short LMT, long RTN when: $\Delta_{norm} > 0.20584$

5. Implementation and Results

Having determined a trading strategy with a set price normalization, strike price, moving average smoothing and TC, as well as a model to run it, we implemented our trading strategy on data running from today's date (01.01.2010) until 31.12.2016. Implementing our strategy in this period resulted in a total of 138 pair trades with our trading capital arbitrarily set at $10,000. Furthermore, we assumed our strategy would be capital neutral, meaning that the revenue from short selling one stock would finance our long position in the other. Such an assumption dismisses the potential collateral requirement when short-selling, but further discussion of this is reserved for later, when discussing the overall limitations of our model. The following results include TC, unless otherwise stated.

The most relevant results are presented in Table 7, where we have calculated the Sharpe ratios, excess return, ROI and P&L (with TC) on an annual, average and cumulative basis. In total, we accumulated a profit of $8,288.81 which constitutes a ROI of 82.89%. Adjusting for alternative cost gave an excess return of 52.79% for the period. Lastly, the strategy gave us a portfolio standard deviation of 0.5386%, as seen in Table 8, and an average daily return on the portfolio ($\overline{r_p}$) of 0.0468%. This gave us an overall Sharpe ratio of 1.015. Overall, all indicators point towards this being a profitable trading strategy.

Year	P&L w/ TC	ROI	Excess return	Sharpe
2010	$2,150.09	21.50%	17.67%	1.963
2011	$2,185.60	21.86%	18.03%	2.158
2012	$458.40	4.58%	0.75%	0.086
2013	$1,342.25	13.42%	9.59%	1.805
2014	$1,593.37	15.93%	12.10%	1.704
2015	$819.10	8.19%	4.36%	0.481
2016	-$260.00	-2.60%	-6.43%	-1.184
Result	**$8,288.81**	**82.89%**	**52.79%**	**1.015**
Average	**$1,184.12**	**11.84%**	**8.01%**	**1.002**

Table 7: Results

Year	Standard deviation
2010	0.6409%
2011	0.5981%
2012	0.6315%
2013	0.3647%
2014	0.4932%
2015	0.6092%
2016	0.3400%
Average	**0.5254%**
Total	**0.5386%**

Table 8: Portfolio Standard Deviations

Figure 5 shows normalized prices and profits from our trading period. The end results show a steady profit growth as the period progresses, interrupted only by intermittent and short-lived trading losses. The normalized prices followed one another, indicating a continuation of the observed correlation in our backtesting period.

Calculations of the portfolio's COE, in accordance with the FF5F model is provided in the appendix. These were done by using factor data collected from Kenneth R. French's webpage (French, 2017). However, our regression analysis exhibited a low coefficient of determination (R^2 ≈ 3%) and p-values for our independent variables mostly larger than 0.05, such that they were not statistically significant. Therefore, we opted to find more reliable numbers by sourcing them from Bloomberg Terminal. Our own calculations gave us a COE of 2.37%, while the more reliable Bloomberg data suggested a COE of 8.9%. Our average annual return of 11.84% was larger than both these numbers, indicating that our pairs portfolio was a better investment than going long in both of these stocks.

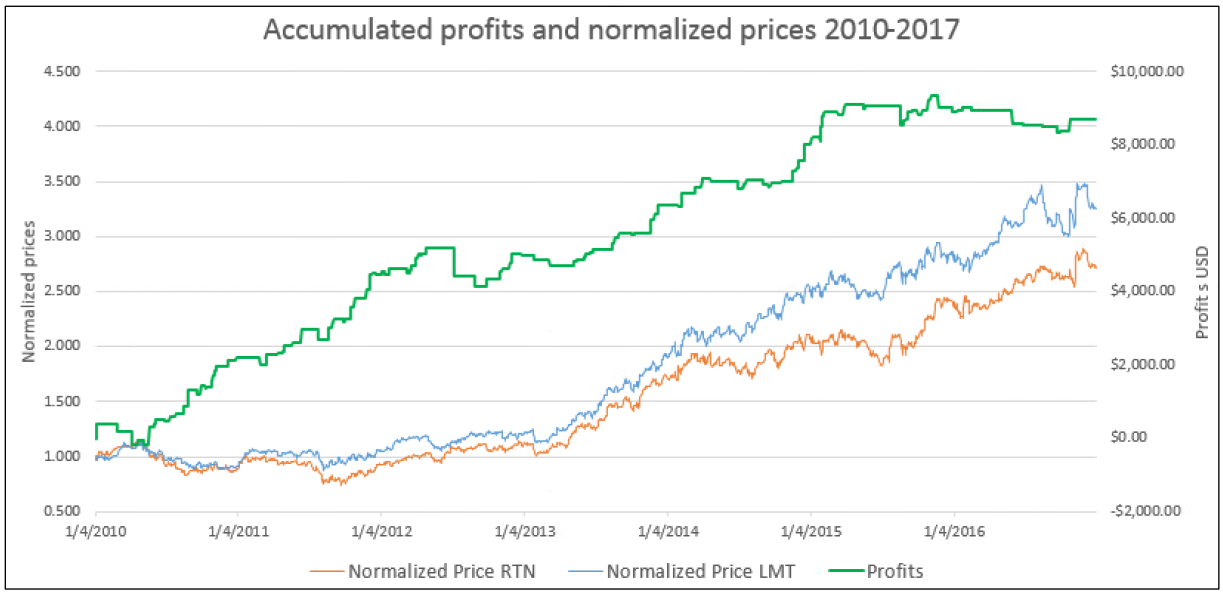


Figure 5: Profits vs Normalized Prices

Even though the total profits were solid, the strategy made a total of 42 unprofitable trades. Furthermore, from Table 7 we see that the yearly profits in 2012 were lower than other years. The normalized prices diverge to some extent from mid-2014 to early 2015, and this could explain why trading profits flatten in 2015 and decline in 2016. This can also be seen in Figure 5, where the profit growth stagnated in these two years. We have researched what the potential causes behind this observed divergence. In March 2014, RTN named a new CEO for the first time in ten years which could cause the stock value to decrease (Raytheon Company, 2014). At the same time LMT acquired two new companies and won a lucrative contract with the Pentagon in June, which could have cause an increase in their stock price (Lockheed Martin, 2017). These changes in the companies could explain the sudden divergence. However, we do not have sufficient statistics to determine a causal relationship between these events.

Analyzing the length of different holding periods give a good indication on how TC varies. Our predetermined stock loan fee of 0.25% annually is compounded as 0.0000068% interest daily for holding short positions. However, as can be seen from Table 9 the average holding period was only 13 trading days, keeping the fee low.

Year	Days held	Days held in %	Trades made	Average holding days
2010	217	86.80%	27	8
2011	227	90.80%	20	11
2012	218	87.20%	20	11
2013	228	91.20%	17	13
2014	227	90.80%	20	11
2015	208	83.20%	24	9
2016	239	95.60%	10	24
Average	**223**	**89.37%**	**20**	**13**
Total	**1564**	-	**138**	-

Table 9: Holding Periods

As one can see from Table 10, the difference between P&L with and without TC is rather small, making little impact on the overall returns.

Year	P&L w/o TC	P&L w/ TC	Difference
2010	$2,217.66	$2,150.09	$67.57
2011	$2,242.03	$2,185.60	$56.43
2012	$517.21	$458.40	$58.81
2013	$1,395.35	$1,342.25	$53.10
2014	$1,653.14	$1,593.37	$59.77
2015	$884.77	$819.10	$65.67
2016	-$219.32	-$260.00	$40.67
Total	**$8,690.82**	**$8,288.81**	**$402.01**

Table 10: P&L with and without TC

A deeper analysis of the results, as shown in Figure 6, illustrates when we reached our strike value of +/- 0.20584. Most of the time the normalized spread moved outside of the strike values [-0.20584, 0.20584], and we held positions as much as 88.76% of the time. When the normalized spread was higher than the orange line (Y=0.20584), we took a long position in RTN and a short position in LMT, and vice versa when the normalized spread was lower than the orange line (Y= -0.20584). Positions were closed when the normalized spread reached 0.

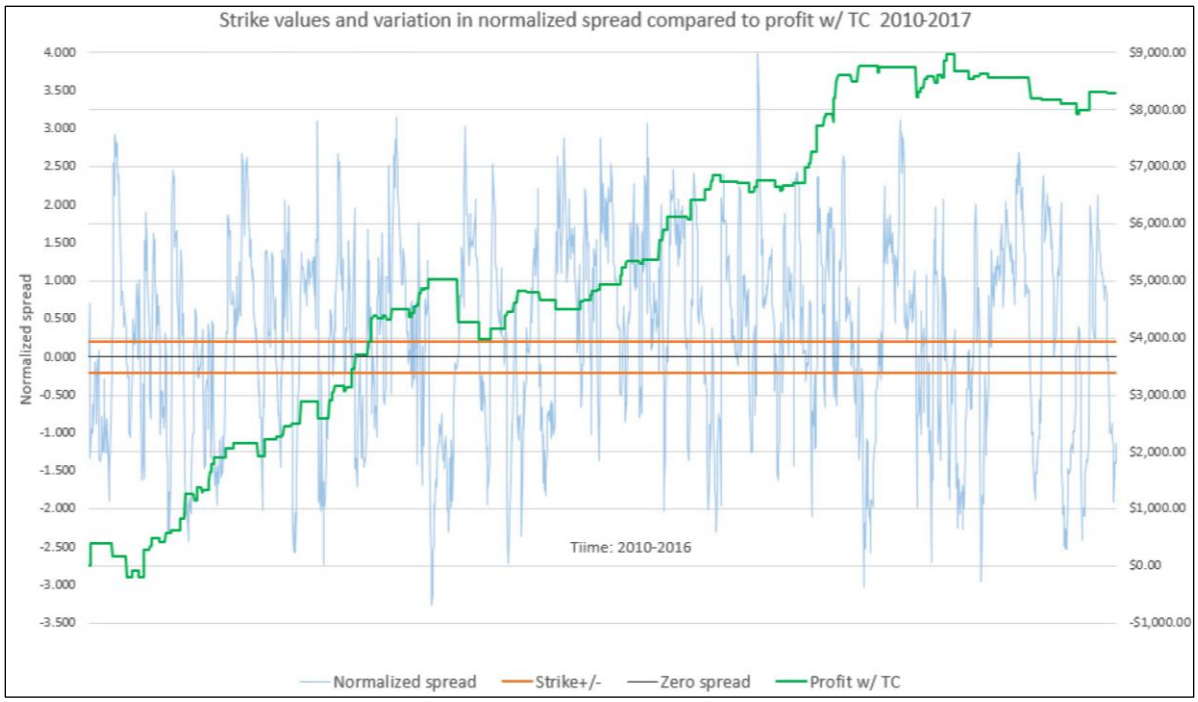


Figure 6: Normalized Spread and Strike Values vs Profits

When backtesting from 2000-2010 the strike value of 0.20584 had the highest Sharpe ratio, as shown in Figure 4. However, this value wouldn't necessarily be the best strike value for the next seven years. Figure 7 shows Sharpe ratio with different strike values for our implementation period. Our strategy clearly lost profit being locked to the 0.20584 strike value. The strike value of 1.8 would have maximized the Sharpe ratio, but this can only be calculated in hindsight through data snooping, such that this strike value could not be implemented. The model is based on backtesting and using the 1.8 strike value from 2000-2010 would be far off the optimal strike value for the backtesting period, therefore this strike value would never have been chosen. This can be seen in Figure 4**,** where a strike value of 1.8 would have given a Sharpe ratio roughly 30% lower for the backtesting period. It is also worth mentioning that the strike value which gave the most profit when backtesting would have given a much lower Sharpe ratio and profit than our strike value of 0.20584.

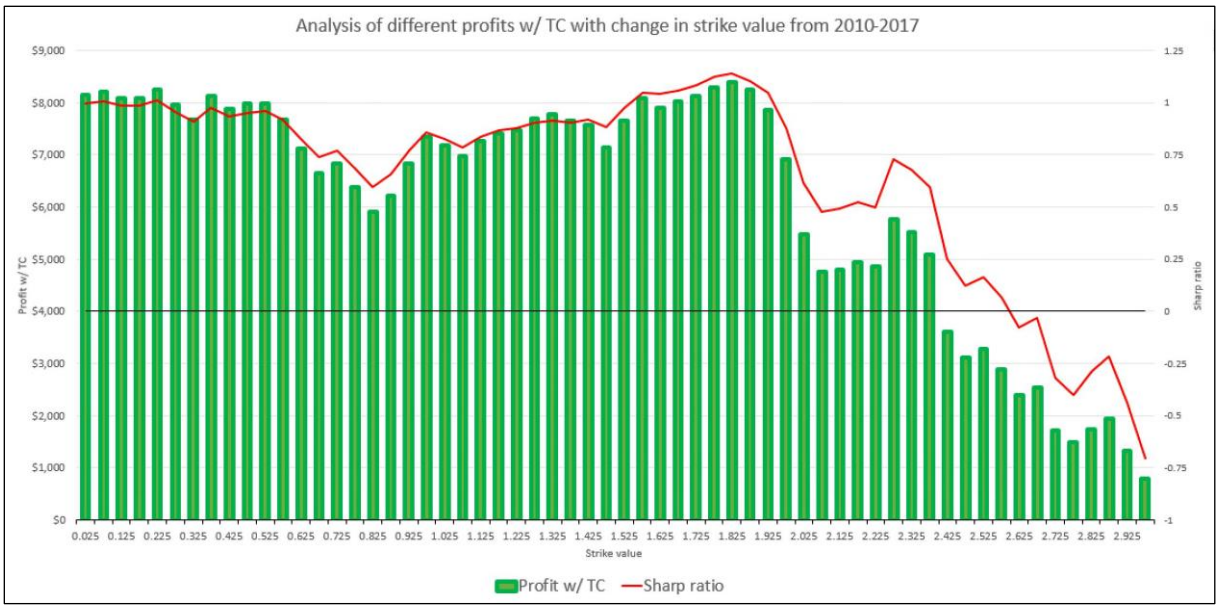


Figure 7: Sharpe Ratio and P&L with different strike values (2010-2017)

6. Constraints, Limitations and Weaknesses

While conducting our analysis and developing our model, we've had to make several assumptions to limit the scope and complexity of our paper. These assumptions, which simplify our scenario, make the trading strategy and model presented in this paper vulnerable to the dynamics of the real world. Furthermore, pairs trading in itself involves several aspects of risk, which may impact the integrity of our findings. In this section of the paper we will present and discuss the most important assumptions, limitations, constraints and weaknesses associated with the pairs trading model we have presented.

Basis risk is the most severe and difficult risk to manage (Harris, 2002), but the model we have developed does not include any measures to counter it. This type of risk is defined as "the risk that the arbitrage basis will move in the wrong direction" (Harris, 2002, p. 267). In our case this would mean that the prices of the stocks diverge permanently. The only way to deal with this would be to implement a stop-loss to minimize our losses. There are many potential causes for such a development, such as changes in the fundamental business model or general decline in one of the companies. A divergence of this kind could also take place on a medium-term outlook if a large investor was to suddenly divest (or invest) in one company, but not the other, thereby significantly changing the price of one company. For example, NBIM, one of the world's largest investors, decided to divest itself of both RTN and LMT in 2005 due to the companies' involvement in the production of cluster munitions (NBIM, 2017). Since the divestment affected both companies, no

significant impact on their price correlations appeared. However, it is plausible that if the divestment decision only affected one of the companies, it would have caused the two companies' stock prices to diverge, negatively impacting pairs traders such as ourselves.

In terms of implementation and liquidity risk, one of our assumptions in this assignment is that trades happen instantly and to the close price. This is not very likely as instant trading requires a trader to put in market order and that will usually force the trader to pay for the spread. In our case the spread will be added or removed from the price we wish to trade at and it's not possible to know how big the impact will be on our final price. If we want to trade to the price we wish we would have to put in a limit order where we specify how many stocks we wish to buy or sell and to which price. There is no certainty that someone will trade at this price or even the amount we require. Failure to trade instantly can lead to both minor and major losses.

It is vital to observe our model's sensitivity to changes in different parameters when evaluating our results. While constructing our model, we selected a stock loan fee, a risk-free rate and a brokerage fee which we believe reflect real-world conditions. However, such parameters are subject to change, and even though we have sources to support our numbers, attainable real-world parameters might be different. Sensitivity analysis conducted on these parameters are shown in the appendix, and indicate that our model is robust to changes in each parameter, but that significant increases in two or more parameters may reduce our risk-adjusted returns to an unprofitable level.

Furthermore, we have assumed that we can trade in non-integer quantities. This assumption allows us to buy stocks for exactly $10,000 every time we wish to do so. Obviously, this is not possible. The assumption causes problem because in reality we would buy more of one stock and less of the other which causes an imbalance in the portfolio. As a result of this our portfolio will not be market neutral as one of the stock is weighted more than the other. This causes exposure to market risk.

When lending stocks, as we do when we short, a collateral usually applies. We have assumed that we don't have to pay such a collateral in our model, which is not realistic. Collateral is similar to a deposit where the borrower pays the lender an agreed upon amount which often surpasses the value of the underlying stock. As this is paid back at the end, the effect on our inventory will probably not be significant, it will however affect how many stocks we are able to short sell. Some of our inventory will be dedicated to pay for collateral and might cause imbalance in our position.

This will affect how market neutral our portfolio will be (InteractiveBrokers, Calculating the Cost of Borrowing Stock at Interactive Brokers, 2017b).

Lastly, we would like to highlight the fact that our thesis question is flawed in itself. By asking whether pairs trading is still profitable, we can easily confirm this by finding simply one profitable trading pair. However, to prove that pairs trading is *not* profitable, we would have had to perform a massive quantitative analysis on every single possible stock pair in our asset universe. This is far beyond the scope of this paper and our abilities. We were lucky that the first pair we tested gave positive results, thereby answering our thesis question and confirming that pairs trading is still profitable, even when adjusted for risk.

7. Conclusion

In this assignment we tried to answer our thesis question: *Is pairs trading still a viable strategy to earn high risk adjusted returns?*
The answer to this question is yes, it is a viable strategy. By backtesting the industry from 2000 to 2010 we found two companies, Raytheon and Lockheed Martin, which in addition to being highly correlated and having similar betas, also were cointegrated. By using inspiration from Reverre we designed a model that gave us a strike value of 0.20548. The strike value was optimized by to maximize the Sharpe ratio, while holding the moving average constant at 30 days. Optimizing this way ensures that risk is considered in the model, which was one of the elements in the thesis question.

After designing the model, we applied it from 2010 to 2017. We traded with a constant sum of $10,000 and ended up with a profit of $8,288.81, which constitutes a total ROI of 82.89%. Our pairs portfolio had a standard deviation of 0.5386%, such that we achieved a Sharpe ratio of 1.015. In total we executed 138 trades where 96 of these were profitable. The average holding days were 13 days, which is not that long considering staying in a short position can be expensive because of the required payments regarding stock loan fees.

All in all, our chosen pair supports the fact that pairs trading still can be profitable, also when adjusting for risk. However, our findings are based on a model and strategy with several simplifications and limitations. Lastly, one cannot determine if pairs trading in general still is profitable based solely on one example of pairs trading.

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9. Appendix

Sensitivity Analyses

Sensitivity analyis changing the different transaction costs and how it has an inpact on the profit w/ TC

Brokerage fee (rows): 0.005%
Stock loan fee (collums): 0.25% p.a
Profit w/ TC: $8,288

	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
0.005%	$7,803.09	$7,184.97	$6,566.46	$5,947.56	$5,328.26	$4,708.57	$4,088.49	$3,468.02	$2,847.15	$2,225.89
0.01%	$7,533.09	$6,914.97	$6,296.46	$5,677.56	$5,058.26	$4,438.57	$3,818.49	$3,198.02	$2,577.15	$1,955.89
0.015%	$7,263.09	$6,644.97	$6,026.46	$5,407.56	$4,788.26	$4,168.57	$3,548.49	$2,928.02	$2,307.15	$1,685.89
0.02%	$6,993.09	$6,374.97	$5,756.46	$5,137.56	$4,518.26	$3,898.57	$3,278.49	$2,658.02	$2,037.15	$1,415.89
0.025%	$6,723.09	$6,104.97	$5,486.46	$4,867.56	$4,248.26	$3,628.57	$3,008.49	$2,388.02	$1,767.15	$1,145.89
0.03%	$6,453.09	$5,834.97	$5,216.46	$4,597.56	$3,978.26	$3,358.57	$2,738.49	$2,118.02	$1,497.15	$875.89
0.035%	$6,183.09	$5,564.97	$4,946.46	$4,327.56	$3,708.26	$3,088.57	$2,468.49	$1,848.02	$1,227.15	$605.89
0.04%	$5,913.09	$5,294.97	$4,676.46	$4,057.56	$3,438.26	$2,818.57	$2,198.49	$1,578.02	$957.15	$335.89
0.045%	$5,643.09	$5,024.97	$4,406.46	$3,787.56	$3,168.26	$2,548.57	$1,928.49	$1,308.02	$687.15	$65.89
0.05%	$5,373.09	$4,754.97	$4,136.46	$3,517.56	$2,898.26	$2,278.57	$1,658.49	$1,038.02	$417.15	-$204.11
0.055%	$5,103.09	$4,484.97	$3,866.46	$3,247.56	$2,628.26	$2,008.57	$1,388.49	$768.02	$147.15	-$474.11
0.06%	$4,833.09	$4,214.97	$3,596.46	$2,977.56	$2,358.26	$1,738.57	$1,118.49	$498.02	-$122.85	-$744.11
0.065%	$4,563.09	$3,944.97	$3,326.46	$2,707.56	$2,088.26	$1,468.57	$848.49	$228.02	-$392.85	-$1,014.11
0.07%	$4,293.09	$3,674.97	$3,056.46	$2,437.56	$1,818.26	$1,198.57	$578.49	-$41.98	-$662.85	-$1,284.11
0.075%	$4,023.09	$3,404.97	$2,786.46	$2,167.56	$1,548.26	$928.57	$308.49	-$311.98	-$932.85	-$1,554.11
0.08%	$3,753.09	$3,134.97	$2,516.46	$1,897.56	$1,278.26	$658.57	$38.49	-$581.98	-$1,202.85	-$1,824.11
0.085%	$3,483.09	$2,864.97	$2,246.46	$1,627.56	$1,008.26	$388.57	-$231.51	-$851.98	-$1,472.85	-$2,094.11
0.09%	$3,213.09	$2,594.97	$1,976.46	$1,357.56	$738.26	$118.57	-$501.51	-$1,121.98	-$1,742.85	-$2,364.11
0.095%	$2,943.09	$2,324.97	$1,706.46	$1,087.56	$468.26	-$151.43	-$771.51	-$1,391.98	-$2,012.85	-$2,634.11
0.10%	$2,673.09	$2,054.97	$1,436.46	$817.56	$198.26	-$421.43	-$1,041.51	-$1,661.98	-$2,282.85	-$2,904.11

Figure 8: Sensitivity Analysis of TC on P&L

Sensitivity analysis changing the different transaction costs and how it has an inpact in the Sharpe ratio:

Brokerage fee (rows): 0.005%
Stock loan fee (collums): 0.25%
Sharp ratio: 1.015

	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
0.005%	0.920	0.801	0.683	0.567	0.454	0.344	0.236	0.132	0.031	-0.066
0.01%	0.873	0.753	0.636	0.521	0.408	0.299	0.192	0.089	-0.011	-0.108
0.015%	0.825	0.706	0.589	0.474	0.362	0.253	0.148	0.045	-0.054	-0.150
0.02%	0.778	0.659	0.542	0.428	0.317	0.208	0.103	0.001	-0.097	-0.191
0.025%	0.730	0.611	0.495	0.381	0.271	0.163	0.059	-0.042	-0.139	-0.233
0.03%	0.682	0.564	0.448	0.335	0.225	0.118	0.015	-0.085	-0.182	-0.274
0.035%	0.633	0.516	0.401	0.288	0.179	0.073	-0.030	-0.129	-0.224	-0.316
0.04%	0.585	0.468	0.354	0.242	0.133	0.028	-0.074	-0.172	-0.266	-0.357
0.045%	0.537	0.420	0.306	0.195	0.088	-0.017	-0.118	-0.215	-0.308	-0.398
0.05%	0.489	0.373	0.259	0.149	0.042	-0.061	-0.161	-0.257	-0.350	-0.439
0.055%	0.440	0.325	0.212	0.103	-0.004	-0.106	-0.205	-0.300	-0.392	-0.479
0.06%	0.392	0.277	0.165	0.056	-0.049	-0.151	-0.249	-0.343	-0.433	-0.519
0.065%	0.343	0.229	0.118	0.010	-0.094	-0.195	-0.292	-0.385	-0.474	-0.559
0.07%	0.295	0.181	0.071	-0.036	-0.140	-0.239	-0.335	-0.427	-0.515	-0.599
0.075%	0.246	0.133	0.024	-0.082	-0.185	-0.283	-0.378	-0.469	-0.556	-0.639
0.08%	0.198	0.086	-0.023	-0.128	-0.230	-0.327	-0.421	-0.511	-0.597	-0.678
0.085%	0.149	0.038	-0.070	-0.174	-0.274	-0.371	-0.464	-0.552	-0.637	-0.718
0.09%	0.101	-0.010	-0.116	-0.220	-0.319	-0.415	-0.506	-0.594	-0.677	-0.757
0.095%	0.053	-0.057	-0.163	-0.265	-0.364	-0.458	-0.548	-0.635	-0.717	-0.795
0.10%	0.004	-0.104	-0.209	-0.311	-0.408	-0.501	-0.590	-0.675	-0.756	-0.834

Figure 9: Sensitivity Analysis of TC on Sharpe Ratio

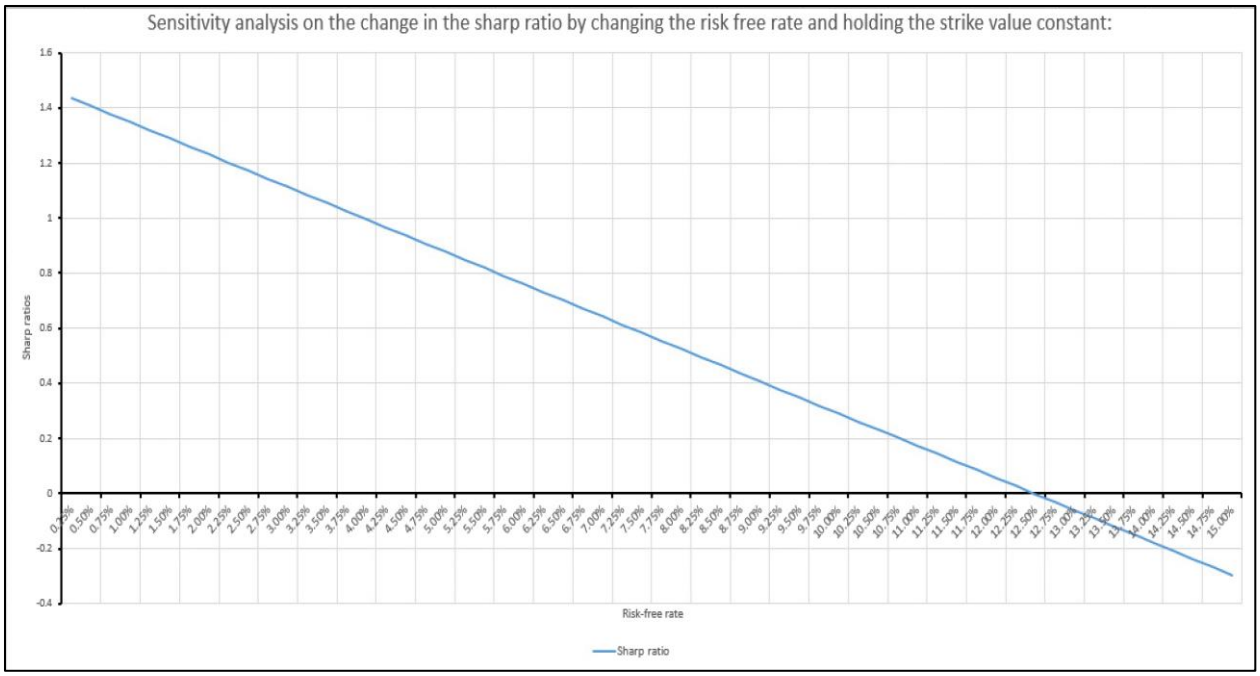


Figure 10: Sensitivity of Sharpe Ratio to Risk-free rate

Engle-Granger Cointegration Test of RTN and LMT Stock Prices

```
Step 1: testing for a unit root in RTN

Augmented Dickey-Fuller test for RTN
including 5 lags of (1-L)RTN
sample size 2509
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0,00194754
  test statistic: tau_c(1) = -1,49583
  asymptotic p-value 0,536
  1st-order autocorrelation coeff. for e: -0,000
  lagged differences: F(5, 2502) = 4,385 [0,0006]

Step 2: testing for a unit root in LMT

Augmented Dickey-Fuller test for LMT
including 5 lags of (1-L)LMT
sample size 2509
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0,00175677
  test statistic: tau_c(1) = -1,77976
  asymptotic p-value 0,3911
  1st-order autocorrelation coeff. for e: -0,000
  lagged differences: F(5, 2502) = 10,474 [0,0000]
```

```
Step 3: cointegrating regression

Cointegrating regression -
OLS, using observations 2000-01-03:2009-12-31 (T = 2515)
Dependent variable: RTN

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const       10,6190      0,184036      57,70    0,0000  ***
  LMT          0,459696    0,00268100   171,5     0,0000  ***

Mean dependent var   40,14481   S.D. dependent var   11,60293
Sum squared resid    26651,79   S.E. of regression   3,256619
R-squared            0,921255   Adjusted R-squared   0,921223
Log-likelihood      -6537,064   Akaike criterion     13078,13
Schwarz criterion    13089,79   Hannan-Quinn         13082,36
rho                  0,982032   Durbin-Watson        0,035913

Step 4: testing for a unit root in uhat

Augmented Dickey-Fuller test for uhat
including 5 lags of (1-L)uhat
sample size 2509
unit-root null hypothesis: a = 1

  model: (1-L)y = (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0,0165916
  test statistic: tau_c(2) = -4,30238
  asymptotic p-value 0,002487
  1st-order autocorrelation coeff. for e: -0,000
  lagged differences: F(5, 2503) = 1,429 [0,2104]

There is evidence for a cointegrating relationship if:
(a) The unit-root hypothesis is not rejected for the individual variables, and
(b) the unit-root hypothesis is rejected for the residuals (uhat) from the
    cointegrating regression.
```

Fama-French Five Factor Model Regression Analysis

Regression Statistics									
Multiple R	0,18423568								
R-squared	0,03394279								
Adjusted R-squared	-0,027984								
Standard Error	0,02681843								
Observations	84								
ANOVA									
	df	*SS*	*MS*	*F*	*Significance F*				
Regression	5	0,00197109	0,00039422	0,54811193	0,73926112			**Risk free rate**	2,38 %
Residuals	78	0,05609978	0,00071923					**Market risk (Return on market - Risk free rate)**	0,0003 %
Total	83	0,05807087						*Market risk premium*	*2,51 %*
								Size risk (Small - big portfolios)	0,02 %
	Coefficients	*Std Error*	*t stat*	*P-values*	*Lower 95%*	*Upper 95%*		*Size premium*	*4,81 %*
Intercept	0,00495553	0,00310358	1,59671368	0,11437332	-0,0012232	0,01113429		**Value risk (Value - growth portfolios)**	-0,18 %
Mkt-RF	3,1457E-06	0,00088227	0,00356551	0,99716424	-0,0017533	0,00175961		*Value premium*	*3,04 %*
SMB	0,00016691	0,00150101	0,11119625	0,91174636	-0,0028214	0,00315519		**Profitiability (Robust - weak portfolios)**	0,12 %
HML	-0,0017695	0,00176626	-1,0018277	0,31952468	-0,0052858	0,00174686		*Profitability premium*	*-1,51 %*
RMW	0,00120354	0,00220976	0,54464721	0,58754999	-0,0031958	0,00560284		**Investment (Conservative - aggresive portfolios)**	-0,05 %
CMA	-0,0005134	0,00282585	-0,1816813	0,85630414	-0,0061392	0,00511244		*Investment premium*	*1,62 %*
Premiums are retrieved from the Fama-French website								**Cost of Equity**	**2,37 %**
URL: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html									

Table 11: FF5F Regression Analysis